# Embracing Cross-Loading to Improve Latent Variable Models Fit: A comparison of available options in SAS®, Mplus™, and R™

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## ABSTRACT

A major criticism of estimating traditional latent variable measurement models is the unrealistic assumption of fixed zero cross-loadings with non-primary factors. Latent variable modeling has evolved from these traditional zero cross-loadings to allowing estimation of non-zero cross-loadings while validating latent constructs of interest. Even though these non-zero cross-loadings are restricted to minimal cross-loadings, they enable researchers to achieve better model fit while allowing a more realistic relationship between items and factors. Both Exploratory Structural Equation Modeling (ESEM) and Bayesian Structural Equation Modeling (BSEM) estimation methods go beyond the traditional model approaches and allow for relaxing restrictive cross-loading assumptions in measurement models. However, different software packages have been slow in adopting these recent developments in latent variable analysis. An extensive literature review of ESEM and BSEM has found estimation availability of both methods only in Mplus and R, with SAS adoption of ESEM, and not BSEM, to date (Muthén & Muthén, 2017; R Core Team, 2022; SAS Institute Inc., 2018). A previous study, which compared the traditional independent cluster model of confirmatory factor analysis (ICM-CFA) approach to ESEM and BSEM, emphasized the improved model estimation with the new flexible ESEM and BSEM approaches in Mplus (Gucciardi & Zyphur, 2016). Using previously collected mental ability test scores data (Holzinger & Swineford, 1939), this study compares the available CFA and ESEM estimation algorithms in SAS to the CFA, ESEM and BSEM estimations algorithms in other software. Results will provide researchers with appropriate application of these methods while evaluating consistency of parameter estimates and model global fit across software packages. Conclusions from this methodological study will highlight how software choice may lead practitioners to making inconsistent decisions about latent variable measurement models. The authors highlight potential areas of growth for SAS PROC CALIS to adopt more recent latent variable modeling options and become more competitive with other software. This could greatly benefit applied researchers and practitioners who use SAS and are interested in adopting more flexible estimation techniques within latent variable modeling and its applications in different fields, especially social science.

## INTRODUCTION

Structural equation modeling (SEM) involves the estimation of latent variables, rather than only measured variables, in a model that includes latent constructs. While estimating latent variables is imperfect by nature, methodological advances in SEM have helped to improve model fit in models that would otherwise have been unidentifiable or poorly fitted with traditional estimation techniques. Confirmatory factor analysis (CFA) was first improved by the development of exploratory structural equation modeling (ESEM), which allowed for cross-loading values to be included in the models, rather than being fixed to zero as is done in traditional CFA (Asparouhov & Muthén, 2009). ESEM allows statistically significant primary cross-loadings to be estimated, while non-primary cross-loadings can still be estimated rather than being fixed to zero. More recently, Bayesian structural equation modeling (BSEM) has sought to further improve model fit by allowing small, non-zero, values for minor cross-loadings to be estimated, rather than fixing them to zero, using prior information (Muthén & Asparouhov, 2012). In other words, researchers can investigate non-primary factor-loadings without facing model identification issues seen in traditional CFA methods (Muthén & Asparouhov, 2012). ESEM and BSEM both offer significant advantages over traditional CFA by

allowing more realistic model hypotheses, and providing better model fit (Koizumi & In'nami, 2020; Muthén & Asparouhov, 2012). While there are many available software options for conducting traditional ICM-CFA, few have been equipped to include both ESEM and BSEM. Even among software packages that have adopted ESEM and BSEM, there is much left to be investigated to ensure the accuracy of estimates provided by individual software algorithms. The purpose of this study is to explore available SAS CFA model estimation methods and compare them to other software capable of estimating both ESEM and BSEM models. Accuracy and precision of these software packages is studied, while providing practitioners with a discussion of the benefits and drawbacks to using SAS for flexible cross-loading estimation methods.

## STRUCTURAL EQUATION MODELING

The emerging field of SEM is valuable because it allows researchers to distinguish between observed and latent variables while directly estimating latent variables and validating latent constructs (Guo et al., 2019). Structural equation models are commonly used in different fields, such as the social sciences, to measure abstract concepts, and therefore typically require some prior domain knowledge to create successful models of latent concepts (Brown, 2015; Guo et al., 2019). In conventional SEM, non-primary factor loadings are fixed to zero and only hypothesized primary factor loadings are freely estimated with non-zero values (Brown, 2015; Muthén & Asparouhov, 2012). ESEM and BSEM techniques have been developed to overcome this unrealistic model specification.

Over the past two decades, ESEM and BSEM techniques have improved model fit and model specification flexibility by allowing non-zero factor loadings, especially non-primary cross-loadings, between items and factors. More recently, BSEM allowed all factors to have non-zero factor loading values, with prior information specification of near zero non-primary factor loadings. BSEM models also specify informative prior distributions (Asparouhov & Muthén, 2021; Muthén & Asparouhov, 2012). Since, it is unlikely that two variables have no association at all, especially when measuring abstract concepts in the social sciences, BSEM may create models which better fit the data (Koizumi & In'nami, 2020). ESEM uses a 2-part approach to estimate models by not fixing non-primary cross-loadings to zero (Asparouhov & Muthén, 2009; Marsh et al., 2009). Part 1 utilizes exploratory factor analysis (EFA) to estimate factor loadings for all primary and non-primary factor loadings of all factors, with a prior hypothesized model for primary factor association to be used as a target population model. The use of a priori hypothesized model allows non-primary cross-loadings to not to be fixed to zero, but rather expected to be close to zero, while primary factor loadings are freely estimated. To summarize, part 1 uses the hypothesized target model specifying primary and non-primary factor association for each item to estimate a model using EFA. For part 2, a CFA model is estimated using the parameter estimates from part 1 as starting values for all items, including non-primary minor cross-loadings (Asparouhov & Muthén, 2009; Marsh et al., 2009). ESEM differs from BSEM in two primary ways: first, while BSEM allows researchers to specify small variance prior information for all model estimates, ESEM does not allow this; second, the optimal loading structure in ESEM is based on the unrotated target exploratory model from part 1, whereas the optimal loading structure in BSEM is determined by all components in the model that are estimated simultaneously (Guo et al., 2019; Muthén & Asparouhov, 2012).

There have been studies, using both simulated and real-world data, that have demonstrated the ability of BSEM and ESEM methods to improve SEM/CFA (Gucciardi & Zyphur, 2016; Guo et al., 2019; Liang et al., 2020; Xiao et al., 2019). Most commonly, BSEM is compared to ICM-CFA and ESEM. Previous research comparing Bayesian and conventional SEM indicated that a Bayesian model with correctly specified informative priors outperformed frequentist models as well as naïve and non-informative Bayesian models (Smid et al., 2019). Although it is not surprising that the use of more accurate prior information outperforms less

informative priors, the accessibility of known prior information in latent variable modeling is quite common. "Prior dependence" is a known issue in parameter estimate bias that practitioners need to consider when defining Bayesian priors (Asparouhov & Muthén, 2012; Muthén & Asparouhov, 2012). Bayesian models have performed better in estimating more complex models because they could incorporate model uncertainty more successfully than traditional SEM methods that involve no prior information (Smid et al., 2019). BSEM models generally have better model fit compared to traditional CFA models, partly due to the Bayesian models being able to determine which covariates should vary and which covariates should remain fixed (Guo et al., 2019). Correctly specifying prior information is crucial to maximize the potential of BSEM models in providing more realistic estimates (Liang, 2020; Smid et al., 2019). It is known that correctly specifying target values is just as crucial to maximizing the potential of ESEM models but BSEM still provides additional model specifications over ESEM (Guo et al., 2019).

There is a growing number of studies that support the notion that ESEM and Bayesian methods applied to SEM improve model fit by allowing for more flexible models. However, this is contingent on the prior information being correctly specified for factor loadings. The primary reason for improvement in model fit is allowing for major and minor, non-zero cross-loadings, rather than only allowing for major non-zero cross-loadings and forcing non-primary cross-loadings to be equal to zero, as is done in traditional SEM (Asparouhov & Muthén, 2009; Marsh et al., 2009; Muthén & Asparouhov, 2012). However, with increased model complexity and model fit may come increased estimation error. There is some concern that the more complex BSEM models, especially those with poor prior specification or non-informative prior distributions, learn sample-specific aspects of the data, and as a result become noisier and less generalizable. Some studies have found that BSEM models have better model fit, but also have higher estimation bias than ESEM models (Guo et al., 2019).

Applied studies using real-world data have similarly found that BSEM and ESEM models have improved model fit over traditional CFA models. Yet comparisons of analyses in Mplus provided varying degrees of model improvement for BSEM over ESEM (Koizumi & In'nami, 2020; Reis, 2017). Improved model fit of BSEM is often attributed to the strength of the known prior information available to researchers. Through all of this research, general software selection by researchers has been relegated to Mplus. There is a need to compare analysis results from Mplus to those from other software packages for traditional, ESEM, and BSEM latent construct estimation/validation methods.

Additionally, there is some documentation of model convergence issues and lengthy computation time when building Bayesian models using the *blavaan* package in R (Merkle & Jorgensen, 2022). The *blavaan* package developers were made aware of a potential issue in early 2022. While any individual can develop an R package, inherent issues are commonly recorded and must be updated by package authors/developers. Authors often become aware of issues through user reported issues and attempt to fix them.

Since the adoption of both BSEM and ESEM methods in different software packages, numerous applied and simulation studies have compared ESEM and BSEM performance, yet they have only utilized Mplus for the analysis (Gucciardi & Zyphur, 2016; Guo et al., 2019; Liang et al., 2020; Xiao et al., 2019). Previous studies have compared parameter estimates for various types of analysis methods across software, but not within SEM framework, resulting in mixed conclusions regarding parameter estimate consistency. While some studies found consistent parameter estimates across software packages for non-SEM analyses, others found unexplainable differences in parameter estimates between different software for non-SEM analysis (Chang et al., 2020; Harper et al., 2011). While consistent parameter estimates across different software is expected, the only way to validate this is to estimate and evaluate models with multiple software. As the literature shows, the previous studies that compared

BSEM and ESEM model parameter estimates used only Mplus for model validation and parameter estimation. None of the studies compared the results of model estimation in Mplus to estimates obtained using R or SAS. An investigation of the available literature performed in August 2022 on *blavaan* R package, package available in R to estimate BSEM, and exploratory factor analysis/ESEM returned only four publications that considered both analysis methods using Mplus and R, and none comparing with SAS. To the best of our knowledge, no comparisons have been done across the aforementioned software, methods, or packages.

## SOFTWARE SELECTION

Documentation for the adoption for ESEM methods in SAS is limited. SAS appears to have adopted the procrustean target rotation in proc factor around 1999 (Raykov & Little, 1999; SAS Institute Inc., 1999) It appears that SAS adopted ESEM during the development of the procedure TCALIS between 2008 and 2011, which aligns with the development of the methodology (Asparouhov & Muthén, 2009; Gu & Wu, 2011; Marsh et al., 2009; Osborne & Banjanovic, 2016; SAS institute Inc., 2008). After an exhaustive search in the literature, to the best of our knowledge, SAS has not adopted BSEM methodology, or does not provide a user-friendly application method for practitioners. The most recent documentation for both PROC CALIS and PROC MCMC do not provide documentation on executing BSEM models in SAS with only path modeling options with no full structural latent variables (Chen et al., 2016; Miočević et al., 2018; SAS Institute Inc., 2018).

The only statistical analysis software that appear to have incorporated both ESEM and BSEM are R and Mplus. BSEM methodology was introduced to Mplus around 2012; R adopted it through the *blavaan* package around 2015 (Merkle & Rosseel, 2015; Muthén & Asparouhov, 2012). ESEM with minor cross-loadings methodology was introduced to Mplus around 2010 and R adopted ESEM within multiple packages including *pysch*, *lavaan*, and *sem* between 2009 and 2013 (Asparouhov & Muthén, 2009; De Beer & Van Zyl, 2019; Guàrdia-Olmos et al., 2013; Muthén & Asparouhov, 2012; Revelle, 2015).

A few studies have compared the parameter estimates of the traditional SEM measurement modeling across different software. No study to our knowledge has compared parameter estimates of BSEM and ESEM models across different software. With the recent introduction of ESEM and BSEM methods into statistical software packages, more research is needed to test the consistency of parameter estimates across different software. As explained before, the primary purpose of this study is to explore parameter estimates and global fit indices across different software that allow for estimation of SEM models with minimal cross-loadings. As mentioned above, to date, only Mplus and R include both ESEM and BSEM methods, while SAS allows for estimation of ESEM models. We ideally hope to find similar parameter estimates across software, as well as similar improvements in model fit in ESEM and BSEM models with minimal cross-loadings, regardless of software choice. We hope this study provides continued support for alternatives to traditional measurement models and the practice of fixing non-primary cross-loadings to zero, while emphasizing the need to include a well documented Bayesian latent variable estimation method into SAS PROC CALIS or PROC MCMC. The results of this study will provide practitioners with knowledge on consistency, or lack of consistency, of parameter estimates when using ESEM and BSEM methodology across software, as well as information on SAS, R, and Mplus software specifications for these modeling techniques.

## METHODS

Analyses were completed in SAS version 9.4 (SAS Institute Inc., 2018), Mplus version 8.6 (Muthén & Muthén, 2017), and R version 4.1.0 (R Core Team, 2022). Within R, the model estimations were completed using the *lavaan* package version 0.6.12 and the *blavaan*

package version 0.4.3, along with each package's various dependencies (Merkle et al., 2021; Rosseel, 2012). As there are various Markov chain Monte Carlo (MCMC) algorithms in Bayesian estimation, all BSEM estimations in *blavaan* were completed using the package default MCMC algorithm, stan, through RSTAN version 2.21.5 (Stan Development Team, 2022). Recent advances in ESEM literature have provided tools to assist practitioners in specifying ESEM models in both R and Mplus. For ease of specifying models with target values in R, the *esemComp* package version 0.2 was used (Silverstrin & De Beer, 2022). For ease of specifying models with target values in Mplus, researchers developed the ESEM Mplus code generator (De Beer & Van Zyl, 2019). These are tools researchers could benefit from to reduce the coding burden required to estimate ESEM models. This is a gap that SAS could fill to provide ease in estimating ESEM models.

## DATASET

The data used for this methodological comparison was obtained from the 1939 Holzinger and Swineford study measuring mental abilities in middle school aged children in 7th and 8th grades at 2 schools (Holzinger & Swineford, 1939). Their dataset has been used throughout the development of modern measurement techniques. 26 different tests were conducted to measure various mental abilities. Each variable was measured on a continuous scale based on the outcome of the various tests conducted by Holzinger and Swineford and were originally hypothesized to measure a general factor and 5 primary factors (Holzinger & Swineford, 1939). Over time, the general factor indicators and two additional indicators have been dropped due to poor fit, leaving 19 of the original indicators to be used in the measurement models (Gustafsson, 2001; Harman, 1976). The same 19 out of the original 26 items were used for analysis in our study replicating original development of BSEM techniques (Muthén & Asparouhov, 2012). Unlike the original analysis by Muthén and Asparouhov, the school group that a participant belonged to was not of interest in this study and excluded in the analysis. Due to the known small sample size convergence issues and possible parameter estimate bias in SEM, all sample participants (n=301) were analyzed in this study rather than separately (Gittner, 2021; Ma, 2020).

## ANALYSIS

A new analysis was completed of the classic Holzinger-Swineford mental abilities dataset (1939), where a simple structure does not fit well (Muthén & Asparouhov, 2012). The original measurement model was modified and proposed to compose four primary factors measuring spatial ability, verbal ability, speed, and memory, which were composed of the 19 items as shown in Figure 1 (Gustafsson, 2001; Harman, 1976; Muthén & Asparouhov, 2012). All non-primary factor loadings were restricted and fixed to zero. Previous studies have found this model to provide unacceptable model fit (Muthén & Asparouhov, 2012). The traditional measurement model with non-primary cross-loadings fixed to zero is provided in Figure 1 and was used as a baseline comparison in this study. Attempts to visualize the flexible approaches of ESEM and BSEM with parameter estimates for item loadings for each factor was not visually appealing and therefore not included.
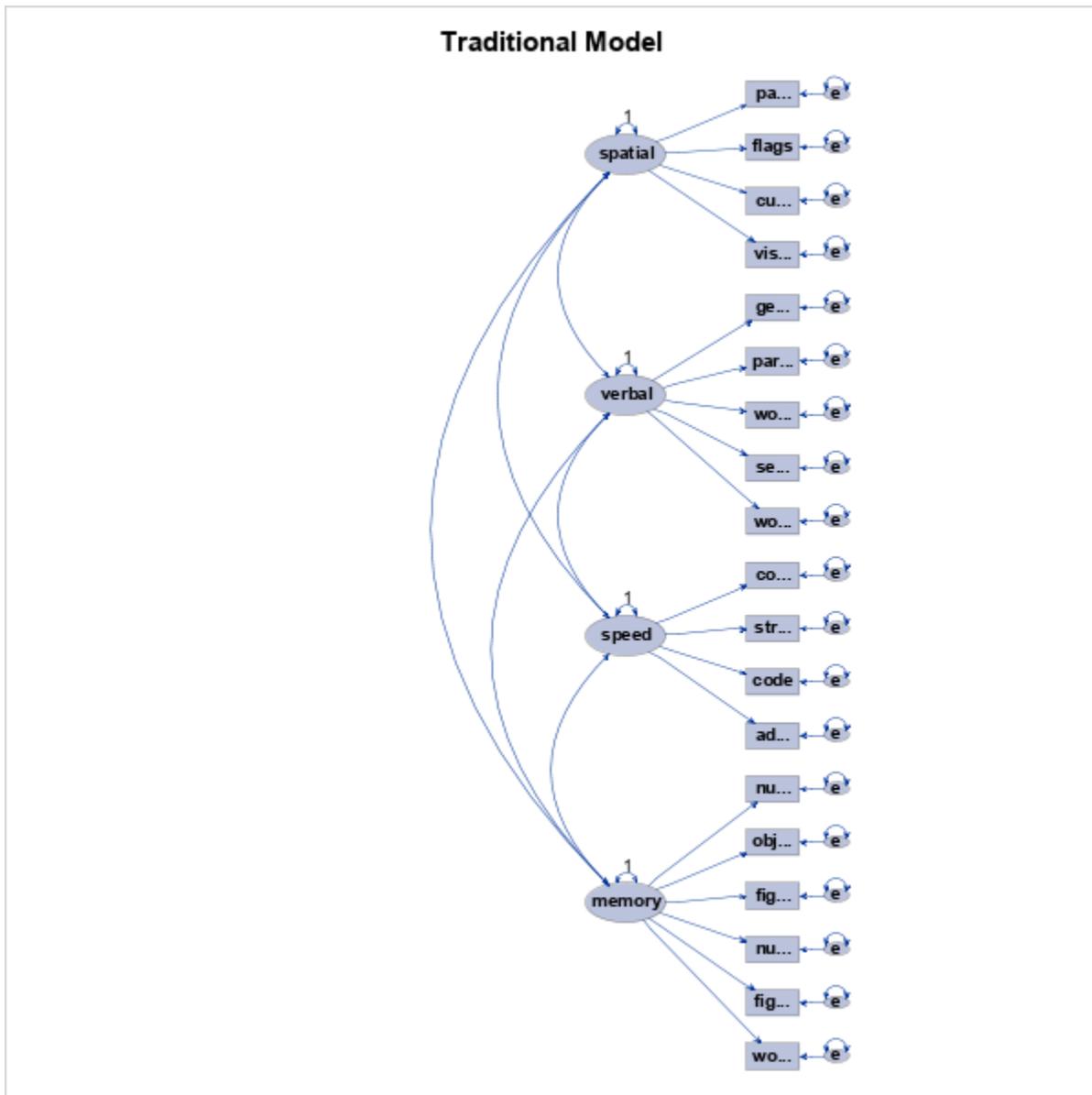
**Figure 1. Hypothesized model: Traditional measurement model where all non-primary cross-loadings are fixed to zero.**

## STUDY MODELS

Three different measurement models were estimated and compared using SAS, R, and Mplus.

Model 1, maximum likelihood (ML), estimated the traditional measurement model with fixed non-primary loadings of zero and a maximum likelihood estimator. Model 1 utilized the previous simple structure model visualized in Figure 1 with no modifications. This model estimated only the X values in Table 1, while fixing all non-primary (NP) loadings to zero. This model was estimated in all 3 software.

Model 2, exploratory structural equation model with non-primary cross-loadings being estimated (ESEM), is comprised of the two-part estimation ESEM-within-CFA approach (Asparouhov & Muthén, 2009; Marsh et al., 2009). Part one utilized the traditional measurement model from model 1 as the target model for primary factors versus non-primary factors that were expected to have small factor loadings close to zero. A target rotation, within

exploratory factor analysis (EFA) was used to estimate factor loadings for items across all factors (Table 1). Part 2 of this model consisted of using the EFA estimates for all primary and non-primary loadings as starting values to estimate the CFA model (Asparouhov & Muthén, 2009; Marsh et al., 2009).

Model 3, Bayesian structural equation modeling with non-primary cross-loadings being estimated (BSEM-CL), adapted the traditional measurement model outlined in Table 1 by eliminating the restriction of fixing the minimal cross-loadings to zero. Instead, informative priors for non-primary factor loadings were defined as N(0, 0.1). This means the prior distribution is assumed to be a normal distribution with mean of zero and standard deviation of 0.1. This model is a more realistic approach compared to model 1 and 2 because it provides an expectation of a minimal relationship between non-primary factors through model specification with a 95% credibility interval of (-.2, .2). All other prior information was specified by the software default settings for BSEM-CL model 3. This model could only be estimated in Mplus and R due to lack of SAS documentation for implementing BSEM.

Models 2 and 3 estimations of all parameters outlined in Table 1, including estimates for all non-primary (NP) loadings that are fixed to zero in model 1's traditional measurement model.

|          | Spatial | Verbal | Speed | Memory |
|----------|---------|--------|-------|--------|
| visual   | X       | NP     | NP    | NP     |
| cubes    | X       | NP     | NP    | NP     |
| paper    | X       | NP     | NP    | NP     |
| flags    | X       | NP     | NP    | NP     |
| general  | NP      | X      | NP    | NP     |
| paragrap | NP      | X      | NP    | NP     |
| sentence | NP      | X      | NP    | NP     |
| wordc    | NP      | X      | NP    | NP     |
| wordm    | NP      | X      | NP    | NP     |
| addition | NP      | NP     | X     | NP     |
| code     | NP      | NP     | X     | NP     |
| counting | NP      | NP     | X     | NP     |
| straight | NP      | NP     | X     | NP     |
| wordr    | NP      | NP     | NP    | X      |
| numberr  | NP      | NP     | NP    | X      |
| figurer  | NP      | NP     | NP    | X      |
| object   | NP      | NP     | NP    | X      |
| numberf  | NP      | NP     | NP    | X      |
| figurew  | NP      | NP     | NP    | X      |

**Table 1. Target rotation for ESEM: X's are primary loadings for items associated with specific factors bolded at the top, NP's are non-primary factor loadings that were only estimated in model 2 and 3. Item names are rows on the left side of the table.**

## ESTIMATION NOTE

All three models were estimated with a standardized latent variable by fixing the latent variable variance to 1.0 (Brown, 2015). Because all indicators were measured on different scales, all observations were standardized before model estimation (Brown, 2015). All models were estimated using a local hard drive on a computer with an Intel i7-9850H processor and 16 GB of RAM.

## Model 1 Traditional ML Estimation

Model 1 in SAS was estimated using PROC CALIS. Model 1 in R was estimated using the *lavaan* package for CFA with the maximum likelihood estimator, and all other package arguments set to the default settings. Model 1 in Mplus used the maximum likelihood estimation with all other software arguments set to the default settings.

Four various modeling languages are available to specify models in PROC CALIS. There are seven various modeling languages available to write SEM models in SAS PROC CALIS. Examples of modeling languages for the FACTOR and LINEQS models in SAS are outlined for both traditional ICM-CFA and ESEM. Both languages will provide the same results. Estimating models using multiple languages ensures that models are defined correctly.

### *Model 1 ML FACTOR Language*

The FACTOR option in PROC CALIS specifies the relationship between factors and items. The PVAR option specifies the factor variances being standardized by fixing each latent variable variance to 1. The COV option specifies the covariance between factors. If using the FACTOR modeling language, model 1 would have been specified as:

```
/*Tradtional CFA approach using FACTOR modeling language*/
proc calis data=import method=ml all outmodel=mlmodel plots=pathdiagram;
   factor
      spatial ---> visual cubes paper flags,
      verbal ---> general paragrap sentence wordc wordm,
      speed ---> addition code counting straight,
      memory ---> wordr numberr figurer object numberf figurew;
   pvar
      spatial = 1,
      verbal = 1,
      speed =1,
      memory = 1;
    cov spatial verbal speed memory;
run;
```

### *Model 1 ML LINEQS Language*

The LINEQS option specifies the intercept and coefficient for each item and its related factor using the EQS modeling language. The VARIANCE option specifies the factor variances being standardized by fixing each latent variable variance to 1. The COV option specifies all factors that covary. The VAR option outlines all items to be included in the model. If using the LINEQS modeling language, model 1 would have been specified as:

```
/*Tradtional CFA approach using LINEQS modeling language*/
proc calis modification data=zimport method=ml all outmodel=mlmodelEQS
      plots=pathdiagram;
   lineqs
     visual = v1 fspatial + e1,
     cubes = v2 fspatial + e2,
     paper = v3 fspatial + e3,
     flags = v4 fspatial + e4,

     general = p1 fverbal + e5,
     paragrap = p2 fverbal + e6,
     sentence = p3 fverbal + e7,
     wordc = p4 fverbal + e8,
     wordm = p5 fverbal + e9,

     addition = q1 fspeed + e10,
     code = q2 fspeed + e11,
```

```
      counting = q3 fspeed + e12,
      straight = q4 fspeed + e13,

      wordr = w1 fmemory + e14,
      numberr = w2 fmemory + e15,
      figurer = w3 fmemory + e16,
      object = w4 fmemory + e17,
      numberf = w5 fmemory + e18,
      figurew = w6 fmemory + e19;

   variance
      fspatial = 1.0,
      fverbal = 1.0,
      fspeed = 1.0,
      fmemory = 1.0,
      e1-e19 = e_var1-e_var19;

   COV
      fspatial fverbal = cspatialverbal,
      fspatial fspeed = cspatialspeed,
      fspatial fmemory = cspatialmemory,
      fverbal fspeed = cverbalspeed,
      fverbal fmemory = cverbalmemory,
      fspeed fmemory = cspeedmemory;

   var visual cubes paper flags general paragrap sentence wordc wordm
      addition code counting straight wordr numberr figurer object numberf
      figurew;
 run;
```

Both model languages estimate the same model; they only use different model specification. PROC CALIS SAS documentation can be reviewed for more details regarding different options.

## Model 2 ESEM Estimation

SAS code to execute model 2 requires a two-part EFA within CFA approach utilizing PROC FACTOR for part 1 and PROC CALIS for part 2. For part 1, the target rotation matrix must first be defined. Then PROC FACTOR will estimate all primary loadings and non-primary factor loadings that are expected to be close to zero, but not fixed to zero with a PROCRUSTES specified target rotation method defined by user. The VAR options outlines all variables to be included in the model.

### Model 2 ESEM Part 1

```
/*part 1 EFA target rotation with maximum likelihood estimation*/
*Define target matrix for rotation in part 1;
data mytarget;
 input _name_ $ visual cubes paper flags general paragrap sentence wordc
   wordm addition code counting straight wordr numberr figurer object
   numberf figurew;
 datalines;
   SPATIAL 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   VERBAL  0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0
   SPEED   0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0
   MEMORY  0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1
;

*ESTIMATE MODEL part 1;
proc factor data=import method=ml NFact=4 rotate=procrustes target=mytarget
```

```
      priors=smc score se plot=pathdiagram;
   var visual cubes paper flags general paragrap sentence wordc wordm
      addition code counting straight wordr numberr figurer object numberf
      figurew;
   run;
```

## Model 2 ESEM Part 2 FACTOR Language

For part 2, the standardized ML parameter estimates from part 1 were used as starting values in estimating the CFA parameters. Some starting values for non-primary items are not contained within parentheses which means the values is fixed; this is necessary to ensure that the model is identifiable. The same item non-primary factor loadings were fixed to the estimates from part 1 across SAS, R, and Mplus to ensure consistency of models. Using this ESEM within CFA approach requires users to specify all standardized coefficients in the FACTOR modeling language. PROC CALIS with FACTOR modeling language for part 2 of ESEM is shown below.

```
/*ESTIMATE part 2 within CFA*/
proc calis data=zimport method=ml all
   outmodel=ESEMmodelP2_1;
  factor
    spatial ---> visual cubes paper flags general paragrap sentence wordc
    wordm addition code counting straight wordr numberr figurer object
    numberf figurew =
    (0.604180294)(0.51466294)(0.463838763)(0.615434152)(-0.017677604)
    (-0.002871533) -0.056916918 (0.068074647) (0.060206344) -0.252715388
    (-0.004310213) (0.091172728) (0.335307811) -0.09429023 (0.041334541)
    (0.278184447) (-0.198021858) (0.05304794) (0.054381722),

    verbal ---> visual cubes paper flags general paragrap sentence wordc
    wordm addition code counting straight wordr numberr figurer object
    numberf figurew =
    (0.11222453) (0.019214669) (0.069609094) -0.139905386 (0.847500244)
    (0.80187633) (0.914298476) (0.690898475) (0.816789116) -0.002946553
    (0.149480738) (-0.069318832) (-0.035547041) 0.07565068 (-0.136605129)
    (0.019259368) (-0.047851096) (-0.007801389) (0.16441266),

    speed ---> visual cubes paper flags general paragrap sentence wordc
    wordm addition code counting straight wordr numberr figurer object
    numberf figurew =
    (0.082812425) (-0.057394568) (0.069235628) 0.074969951 (0.057163241)
    (-0.016231669) -0.001521726 (0.026018463) (-0.030763977) (0.7525504)
    (0.532470144) (0.720272162) (0.555910341) -0.148792119        (-
    0.099551618) (-0.030492079) (0.238004067) (0.134974098) (0.018630109),

    memory ---> visual cubes paper flags general paragrap sentence wordc
    wordm addition code counting straight wordr numberr figurer object
    numberf figurew =
    (0.066394606) (-0.013717339) (-0.071229942) 0.130528342 (-0.080556407)
    (0.086601124) -0.052985295 (0.052096828) (0.041199379) 0.053021061
    (0.170078121) (-0.090945782) (-0.038455025) (0.709347033) (0.636190076)
    (0.492339507) (0.573712653) (0.433723732) (0.332903558);
  Pvar
    spatial = 1, verbal = 1, speed =1, memory = 1;
  cov spatial verbal speed memory;
  VAR visual cubes paper flags general paragrap sentence wordc wordm
    addition code counting straight wordr numberr figurer object numberf
    figurew;
  run;
```

### Model 2 ESEM Part 2 LINEQS Language

Using this ESEM within CFA approach requires users to specify all regression coefficients in the LINEQS modeling language. PROC CALIS with LINEQS modeling language for part 2 of ESEM is shown below.

```
/*ESTIMATE part 2 within CFA*/
proc calis modification data=zimport method=ml all outmodel=mlmodelEQS
    plots=pathdiagram;
LINEQS
visual = (.60418)     fspatial + (0.11222) fverbal + (0.08281) fspeed +
          (0.06639)   fmemory+ e1,
cubes =   (0.51466)   fspatial + (0.01921) fverbal +(-0.05739) fspeed +
          (-0.01372)  fmemory+ e2,
paper =   (0.46384)   fspatial + (0.06961) fverbal +(0.06924) fspeed  +
          (-0.07123)  fmemory+ e3,
/*flags non primary loadings fixed to ensure model identification in ESEM
approach*/
flags =   (0.61543)   fspatial - 0.13991 fverbal + 0.07497 fspeed + 0.13053
                      fmemory+ e4,
general = (-0.01768) fspatial + (0.84750) fverbal + (0.05716) fspeed +
          (-0.08056)  fmemory+ e5,
paragrap = (-0.00287)fspatial + (0.80188) fverbal  + (-0.01623) fspeed +
          (0.08660)   fmemory+ e6,
/*sentence non primary loadings fixed to ensure model identification in
ESEM approach*/
sentence = -0.05692  fspatial + (0.91430) fverbal - 0.00152 fspeed -
          0.05299     fmemory+ e7,
wordc =   (0.06807)   fspatial +  (0.69090) fverbal + (0.02602) fspeed
    +     (0.05210)   fmemory+ e8,
wordm =   (0.06021)   fspatial +  (0.81679) fverbal + (-0.03076) fspeed
    +     (0.04120)   fmemory+ e9,
/*addition non primary loadings fixed to ensure model identification in
ESEM approach*/
addition =  -0.25272  fspatial - 0.00295 fverbal + (0.75255) fspeed +
          0.05302     fmemory+ e10,
code =    (-0.00431)  fspatial + (0.14948) fverbal + (0.53247) fspeed +
          (0.17008)   fmemory+ e11,
counting = (0.09117) fspatial + (-0.06932) fverbal + (0.72027) fspeed +
          (-0.09095)  fmemory+ e12,
straight = (0.33531) fspatial + (-0.03555) fverbal + (0.55591) fspeed +
          (-0.03846)  fmemory+ e13,
/*wordr non primary loadings fixed to ensure model identification in ESEM
approach*/
wordr =   -0.09429    fspatial + 0.07565 fverbal - 0.14879 fspeed +
          (0.70935)   fmemory+ e14,
numberr = (0.04133)  fspatial + (-0.13661) fverbal + (-0.09955) fspeed +
          (0.63619)   fmemory+ e15,
figurer = (0.27818)  fspatial + (0.01926) fverbal + (-0.03049) fspeed +
          (0.49234)   fmemory+ e16,
object =  (-0.19802)  fspatial +   (-0.04785) fverbal + (0.23800) fspeed +
          (0.57371)   fmemory+ e17,
numberf = (0.05305)  fspatial + (-0.00780) fverbal + (0.13497) fspeed +
          (0.43372)   fmemory+ e18,
figurew = (0.05438)  fspatial + (0.16441) fverbal + (0.01863) fspeed +
          (0.33290)   fmemory+ e19;

    variance
        fspatial = 1.0,
```

```
      fverbal = 1.0,
      fspeed = 1.0,
      fmemory = 1.0,
      e1-e19 = e_var1-e_var19;

  COV
      fspatial fverbal = cspatialverbal,
      fspatial fspeed = cspatialspeed,
      fspatial fmemory = cspatialmemory,
      fverbal fspeed = cverbalspeed,
      fverbal fmemory = cverbalmemory,
      fspeed fmemory = cspeedmemory;

  var visual cubes paper flags general paragrap sentence wordc wordm
      addition code counting straight wordr numberr figurer object numberf
      figurew;
  run;
```

Specification of measurement models using FACTOR and LINEQS language is slightly different. LINEQS language is a system of equations with all factors and associated started values being outlined for each item based on EQS programming language. FACTOR language specifies the associated starting value for each item as it's related to the factor but not as a system of equations, as seen above.

## Model 3 BSEM Estimation

Estimation of models using BSEM was only available in R and Mplus. R used the *blavaan* package using the bcfa() function with the STAN MCMC sampling method, while Mplus utilized the Bayes estimator with Mplus' default MCMC sampling method. The number of MCMC iterations was set to 10,000 for both software for the BSEM estimations. The first half of the iterations (n=5,000) were designated as burn-in iterations and the last half of the samples (n=5,000) were used for estimation. Two MCMC chains were used for the BSEM models. While a more appropriate technique may have used the potential scale reduction (PSR) value to provide evidence of convergence towards equilibrium of Bayesian chains. Using this method may result in different iterations being completed across software. Instead, the PSR values of each Bayesian model were inspected after the 10,000 burn-in and sample iterations for PSR values below 0.05, suggesting that all chains converged. Using R STAN's default starting values for the BSEM estimates presented an issue with model convergence in R. Therefore, the starting values were specified to match the Mplus starting values. The initial starting values in *blavaan*'s STAN and in Mplus were chosen as if the maximum likelihood approach was being used to set starting values.

All primary factor loadings were estimated using software default's non-informative prior information and non-primary cross-loadings were estimated using informative priors specified with a mean/variance relationship of N(0,0.1). All other software options were set to the default settings and default non-informative priors for both R and Mplus. R required an additional null model to be estimated, which was used in calculating additional global fit statistics.

## RESULTS

The results of this study are presented as a comparison of the standardized factor loading estimates between the SAS, R, and Mplus, for each of the three models in Tables 2 through 6. Global fit indices then were estimated and compared between SAS, R, and Mplus, for each of the three models in Table 7. Additional global fit indices available for BSEM estimation in R were reported in Table 8. SAS ESEM global fit indices are reported in Table 9 Finally, the

estimation times were reported in Table 10 for all three models in SAS, R, and Mplus.

## STANDARDIZED FACTOR LOADINGS MODEL 1

| | SAS ML estimation | | | | Mplus ML estimation | | | |
|---|---|---|---|---|---|---|---|---|
| | Spatial | Verbal | Speed | Memory | Spatial | Verbal | Speed | Memory |
| visual | 0.749 (0.045) | 0 | 0 | 0 | 0.750 (0.046) | 0 | 0 | 0 |
| cubes | 0.434 (0.057) | 0 | 0 | 0 | 0.434 (0.058) | 0 | 0 | 0 |
| paper | 0.491 (0.055) | 0 | 0 | 0 | 0.491 (0.055) | 0 | 0 | 0 |
| flags | 0.605 (0.050) | 0 | 0 | 0 | 0.605 (0.051) | 0 | 0 | 0 |
| general | 0 | 0.836 (0.021) | 0 | 0 | 0 | 0.836 (0.021) | 0 | 0 |
| paragrap | 0 | 0.821 (0.022) | 0 | 0 | 0 | 0.821 (0.022) | 0 | 0 |
| sentence | 0 | 0.867 (0.018) | 0 | 0 | 0 | 0.867 (0.018) | 0 | 0 |
| wordc | 0 | 0.741 (0.029) | 0 | 0 | 0 | 0.741 (0.029) | 0 | 0 |
| wordm | 0 | 0.847 (0.020) | 0 | 0 | 0 | 0.847 (0.020) | 0 | 0 |
| addition | 0 | 0 | 0.585 (0.048) | 0 | 0 | 0 | 0.585 (0.049) | 0 |
| code | 0 | 0 | 0.718 (0.040) | 0 | 0 | 0 | 0.718 (0.042) | 0 |
| counting | 0 | 0 | 0.626 (0.045) | 0 | 0 | 0 | 0.626 (0.047) | 0 |
| straight | 0 | 0 | 0.678 (0.042) | 0 | 0 | 0 | 0.678 (0.044) | 0 |
| wordr | 0 | 0 | 0 | 0.578 (0.050) | 0 | 0 | 0 | 0.578 (0.051) |
| numberr | 0 | 0 | 0 | 0.517 (0.053) | 0 | 0 | 0 | 0.517 (0.054) |
| figurer | 0 | 0 | 0 | 0.604 (0.048) | 0 | 0 | 0 | 0.604 (0.050) |
| object | 0 | 0 | 0 | 0.556 (0.051) | 0 | 0 | 0 | 0.556 (0.052) |
| numberf | 0 | 0 | 0 | 0.548 (0.051) | 0 | 0 | 0 | 0.548 (0.053) |
| figurew | 0 | 0 | 0 | 0.454 (0.056) | 0 | 0 | 0 | 0.454 (0.057) |

**Table 2. Comparing standardized factor loading estimates for model 1 ML traditional CFA in SAS with Mplus: Standardized factor loadings and (standard errors) for all parameter estimates are reported here.**

| | SAS ML estimation | | | | R ML estimation (lavaan) | | | |
|---|---|---|---|---|---|---|---|---|
| | Spatial | Verbal | Speed | Memory | Spatial | Verbal | Speed | Memory |
| visual | 0.749 (0.045) | 0 | 0 | 0 | 0.749 (0.045) | 0 | 0 | 0 |
| cubes | 0.434 (0.057) | 0 | 0 | 0 | 0.434 (0.057) | 0 | 0 | 0 |
| paper | 0.491 (0.055) | 0 | 0 | 0 | 0.491 (0.055) | 0 | 0 | 0 |
| flags | 0.605 (0.050) | 0 | 0 | 0 | 0.605 (0.050) | 0 | 0 | 0 |
| general | 0 | 0.836 (0.021) | 0 | 0 | 0 | 0.836 (0.021) | 0 | 0 |
| paragrap | 0 | 0.821 (0.022) | 0 | 0 | 0 | 0.821 (0.022) | 0 | 0 |
| sentence | 0 | 0.867 (0.018) | 0 | 0 | 0 | 0.867 (0.018) | 0 | 0 |
| wordc | 0 | 0.741 (0.029) | 0 | 0 | 0 | 0.741 (0.029) | 0 | 0 |
| wordm | 0 | 0.847 (0.020) | 0 | 0 | 0 | 0.847 (0.020) | 0 | 0 |
| addition | 0 | 0 | 0.585 (0.048) | 0 | 0 | 0 | 0.585 (0.047) | 0 |
| code | 0 | 0 | 0.718 (0.040) | 0 | 0 | 0 | 0.718 (0.040) | 0 |
| counting | 0 | 0 | 0.626 (0.045) | 0 | 0 | 0 | 0.626 (0.045) | 0 |
| straight | 0 | 0 | 0.678 (0.042) | 0 | 0 | 0 | 0.678 (0.042) | 0 |
| wordr | 0 | 0 | 0 | 0.578 (0.050) | 0 | 0 | 0 | 0.578 (0.050) |
| numberr | 0 | 0 | 0 | 0.517 (0.053) | 0 | 0 | 0 | 0.517 (0.053) |
| figurer | 0 | 0 | 0 | 0.604 (0.048) | 0 | 0 | 0 | 0.604 (0.048) |
| object | 0 | 0 | 0 | 0.556 (0.051) | 0 | 0 | 0 | 0.556 (0.051) |
| numberf | 0 | 0 | 0 | 0.548 (0.051) | 0 | 0 | 0 | 0.548 (0.051) |
| figurew | 0 | 0 | 0 | 0.454 (0.056) | 0 | 0 | 0 | 0.454 (0.056) |

**Table 3. Comparing standardized factor loading estimates for model 1 ML traditional CFA in SAS with R: Standardized factor loadings and (standard errors) for all parameter estimates are reported here.**

**STANDARDIZED FACTOR LOADINGS MODEL 2**

| | SAS ESEM | | | | Mplus ESEM | | | |
|---|---|---|---|---|---|---|---|---|
| | Spatial | Verbal | Speed | Memory | Spatial | Verbal | Speed | Memory |
| visual | 0.604 (0.065) | 0.112 (0.073) | 0.083 (0.081) | 0.066 (0.085) | 0.587 (0.063) | 0.147 (0.071) | 0.089 (0.080) | 0.082 (0.084) |
| cubes | 0.515 (0.071) | 0.019 (0.074) | -0.057 (0.084) | -0.014 (0.086) | 0.499 (0.068) | 0.046 (0.071) | -0.052 (0.082) | 0.000 (0.084) |
| paper | 0.464 (0.072) | 0.070 (0.073) | 0.069 (0.082) | -0.071 (0.086) | 0.449 (0.069) | 0.096 (0.071) | 0.073 (0.081) | -0.058 (0.085) |
| flags | 0.615 (0.050) | 0.140 (0.006) | 0.075 (0.003) | 0.131 (0.005) | 0.599 (0.051) | -0.105 (0.004) | 0.081 (0.003) | 0.147 (0.006) |
| general | -0.018 (0.057) | 0.847 (0.033) | 0.057 (0.055) | -0.081 (0.058) | -0.017 (0.057) | 0.847 (0.033) | 0.058 (0.056) | -0.083 (0.059) |
| paragrap | -0.003 (0.058) | 0.802 (0.035) | -0.016 (0.056) | 0.087 (0.058) | -0.001 (0.056) | 0.801 (0.033) | -0.013 (0.056) | 0.084 (0.057) |
| sentence | 0.057 (0.002) | 0.914 (0.018) | 0.002 (0.000) | 0.053 (0.002) | -0.055 (0.002) | 0.911 (0.019) | 0.000 (0.000) | -0.057 (0.002) |
| wordc | 0.068 (0.062) | 0.691 (0.042) | 0.026 (0.061) | 0.052 (0.063) | 0.067 (0.059) | 0.695 (0.040) | 0.029 (0.060) | 0.052 (0.062) |
| wordm | 0.060 (0.056) | 0.817 (0.033) | -0.031 (0.055) | 0.041 (0.057) | 0.059 (0.056) | 0.819 (0.032) | -0.028 (0.056) | 0.040 (0.058) |
| addition | 0.253 (0.010) | 0.003 (0.000) | 0.753 (0.049) | 0.053 (0.002) | -0.241 (0.010) | -0.004 (0.000) | 0.746 (0.051) | 0.046 (0.002) |
| code | -0.004 (0.075) | 0.149 (0.063) | 0.532 (0.061) | 0.170 (0.072) | 0.000 (0.073) | 0.158 (0.062) | 0.530 (0.061) | 0.169 (0.073) |
| counting | 0.091 (0.081) | -0.069 (0.071) | 0.720 (0.064) | -0.091 (0.085) | 0.091 (0.079) | -0.052 (0.069) | 0.716 (0.063) | -0.087 (0.084) |
| straight | 0.335 (0.076) | -0.036 (0.073) | 0.556 (0.069) | -0.038 (0.086) | 0.327 (0.077) | -0.008 (0.072) | 0.555 (0.072) | -0.029 (0.089) |
| wordr | 0.094 (0.004) | 0.076 (0.003) | 0.149 (0.006) | 0.709 (0.050) | -0.084 (0.003) | 0.069 (0.003) | -0.144 (0.006) | 0.702 (0.050) |
| numberr | 0.041 (0.085) | -0.137 (0.073) | -0.100 (0.084) | 0.636 (0.071) | 0.047 (0.082) | -0.135 (0.071) | -0.095 (0.083) | 0.634 (0.070) |
| figurer | 0.278 (0.076) | 0.019 (0.069) | -0.030 (0.078) | 0.492 (0.070) | 0.275 (0.073) | 0.035 (0.067) | -0.024 (0.077) | 0.497 (0.069) |
| object | -0.198 (0.084) | -0.048 (0.071) | 0.238 (0.080) | 0.574 (0.071) | -0.184 (0.083) | -0.054 (0.069) | 0.238 (0.080) | 0.565 (0.072) |
| numberf | 0.053 (0.080) | -0.008 (0.069) | 0.135 (0.076) | 0.434 (0.071) | 0.057 (0.080) | -0.002 (0.068) | 0.137 (0.078) | 0.432 (0.074) |
| figurew | 0.054 (0.078) | 0.164 (0.067) | 0.019 (0.076) | 0.333 (0.074) | 0.057 (0.077) | 0.168 (0.065) | 0.022 (0.076) | 0.332 (0.074) |

**Table 4. Comparing standardized factor loading estimates for model 2 ESEM in SAS with Mplus: Standardized factor loadings and (standard errors) for all parameter estimates are reported here.**

| | SAS ESEM | | | | R ESEM | | | |
|---|---|---|---|---|---|---|---|---|
| | Spatial | Verbal | Speed | Memory | Spatial | Verbal | Speed | Memory |
| visual | 0.604 (0.065) | 0.112 (0.073) | 0.083 (0.081) | 0.066 (0.085) | 0.587 (0.062) | 0.146 (0.071) | 0.089 (0.080) | 0.082 (0.084) |
| cubes | 0.515 (0.071) | 0.019 (0.074) | -0.057 (0.084) | -0.014 (0.086) | 0.499 (0.068) | 0.046 (0.072) | -0.052 (0.083) | 0.001 (0.084) |
| paper | 0.464 (0.072) | 0.070 (0.073) | 0.069 (0.082) | -0.071 (0.086) | 0.449 (0.069) | 0.096 (0.071) | 0.073 (0.081) | -0.058 (0.085) |
| flags | 0.615 (0.050) | 0.140 (0.006) | 0.075 (0.003) | 0.131 (0.005) | 0.598 (0.051) | -0.105 (0.004) | 0.081 (0.003) | 0.147 (0.006) |
| general | -0.018 (0.057) | 0.847 (0.033) | 0.057 (0.055) | -0.081 (0.058) | -0.017 (0.055) | 0.847 (0.032) | 0.059 (0.054) | -0.083 (0.057) |
| paragrap | -0.003 (0.058) | 0.802 (0.035) | -0.016 (0.056) | 0.087 (0.058) | -0.001 (0.056) | 0.801 (0.033) | -0.013 (0.056) | 0.083 (0.057) |
| sentence | 0.057 (0.002) | 0.914 (0.018) | 0.002 (0.000) | 0.053 (0.002) | -0.055 (0.002) | 0.911 (0.018) | 0.000 (0.000) | -0.057 (0.002) |
| wordc | 0.068 (0.062) | 0.691 (0.042) | 0.026 (0.061) | 0.052 (0.063) | 0.067 (0.059) | 0.695 (0.040) | 0.029 (0.060) | 0.052 (0.062) |
| wordm | 0.060 (0.056) | 0.817 (0.033) | -0.031 (0.055) | 0.041 (0.057) | 0.059 (0.054) | 0.819 (0.031) | -0.027 (0.055) | 0.040 (0.056) |
| addition | 0.253 (0.010) | 0.003 (0.000) | 0.753 (0.049) | 0.053 (0.002) | -0.241 (0.010) | -0.004 (0.000) | 0.746 (0.049) | 0.046 (0.002) |
| code | -0.004 (0.075) | 0.149 (0.063) | 0.532 (0.061) | 0.170 (0.072) | 0.000 (0.072) | 0.158 (0.062) | 0.530 (0.060) | 0.169 (0.071) |
| counting | 0.091 (0.081) | -0.069 (0.071) | 0.720 (0.064) | -0.091 (0.085) | 0.090 (0.078) | -0.053 (0.069) | 0.716 (0.063) | -0.087 (0.083) |
| straight | 0.335 (0.076) | -0.036 (0.073) | 0.556 (0.069) | -0.038 (0.086) | 0.327 (0.073) | -0.008 (0.071) | 0.555 (0.067) | -0.028 (0.085) |
| wordr | 0.094 (0.004) | 0.076 (0.003) | 0.149 (0.006) | 0.709 (0.050) | -0.084 (0.003) | 0.069 (0.003) | -0.144 (0.006) | 0.702 (0.050) |
| numberr | 0.041 (0.085) | -0.137 (0.073) | -0.100 (0.084) | 0.636 (0.071) | 0.047 (0.082) | -0.135 (0.071) | -0.095 (0.083) | 0.634 (0.070) |
| figurer | 0.278 (0.076) | 0.019 (0.069) | -0.030 (0.078) | 0.492 (0.070) | 0.275 (0.073) | 0.035 (0.067) | -0.024 (0.077) | 0.497 (0.069) |
| object | -0.198 (0.084) | -0.048 (0.071) | 0.238 (0.080) | 0.574 (0.071) | -0.185 (0.081) | -0.054 (0.069) | 0.238 (0.079) | 0.565 (0.070) |
| numberf | 0.053 (0.080) | -0.008 (0.069) | 0.135 (0.076) | 0.434 (0.071) | 0.057 (0.077) | -0.002 (0.067) | 0.137 (0.075) | 0.433 (0.070) |
| figurew | 0.054 (0.078) | 0.164 (0.067) | 0.019 (0.076) | 0.333 (0.074) | 0.057 (0.075) | 0.168 (0.065) | 0.022 (0.075) | 0.332 (0.072) |

**Table 5. Comparing standardized factor loading estimates for model 2 ESEM in SAS with R: Standardized factor loadings and (standard errors) for all parameter estimates are reported here.**

**STANDARDIZED FACTOR LOADINGS MODEL 3**

| | Mplus BSEM estimation | | | | R BSEM estimation | | | |
|---|---|---|---|---|---|---|---|---|
| | Spatial | Verbal | Speed | Memory | Spatial | Verbal | Speed | Memory |
| visual | 0.637 (0.074) | 0.089 (0.066) | 0.050 (0.070) | 0.038 (0.071) | 0.631 (0.070) | 0.097 (0.066) | 0.055 (0.069) | 0.045 (0.070) |
| cubes | 0.510 (0.076) | 0.011 (0.063) | -0.052 (0.066) | -0.023 (0.068) | 0.505 (0.073) | 0.017 (0.063) | -0.049 (0.067) | -0.021 (0.068) |
| paper | 0.483 (0.074) | 0.049 (0.063) | 0.025 (0.066) | -0.055 (0.067) | 0.480 (0.073) | 0.054 (0.062) | 0.029 (0.066) | -0.050 (0.068) |
| flags | 0.639 (0.073) | -0.109 (0.066) | 0.036 (0.069) | 0.077 (0.072) | 0.635 (0.070) | -0.103 (0.067) | 0.041 (0.070) | 0.084 (0.070) |
| general | -0.030 (0.060) | 0.860 (0.041) | 0.040 (0.058) | -0.074 (0.061) | -0.026 (0.060) | 0.858 (0.040) | 0.039 (0.058) | -0.073 (0.058) |
| paragrap | 0.001 (0.059) | 0.809 (0.041) | -0.014 (0.058) | 0.057 (0.059) | 0.005 (0.058) | 0.808 (0.040) | -0.016 (0.058) | 0.058 (0.059) |
| sentence | -0.068 (0.061) | 0.925 (0.042) | -0.006 (0.060) | -0.056 (0.062) | -0.063 (0.060) | 0.922 (0.038) | -0.007 (0.059) | -0.055 (0.060) |
| wordc | 0.041 (0.059) | 0.708 (0.045) | 0.021 (0.058) | 0.041 (0.059) | 0.044 (0.058) | 0.707 (0.044) | 0.018 (0.058) | 0.045 (0.058) |
| wordm | 0.046 (0.059) | 0.831 (0.040) | -0.036 (0.057) | 0.029 (0.058) | 0.048 (0.058) | 0.830 (0.039) | -0.038 (0.057) | 0.031 (0.058) |
| addition | -0.192 (0.073) | -0.020 (0.071) | 0.756 (0.071) | 0.014 (0.075) | -0.190 (0.072) | -0.023 (0.070) | 0.755 (0.069) | 0.011 (0.073) |
| code | -0.005 (0.065) | 0.114 (0.063) | 0.585 (0.067) | 0.113 (0.067) | -0.003 (0.064) | 0.115 (0.061) | 0.584 (0.065) | 0.113 (0.067) |
| counting | 0.041 (0.071) | -0.052 (0.066) | 0.716 (0.068) | -0.072 (0.070) | 0.044 (0.071) | -0.055 (0.067) | 0.720 (0.067) | -0.075 (0.071) |
| straight | 0.226 (0.066) | -0.013 (0.063) | 0.581 (0.073) | -0.027 (0.069) | 0.228 (0.067) | -0.012 (0.063) | 0.585 (0.071) | -0.029 (0.068) |
| wordr | -0.082 (0.072) | 0.040 (0.067) | -0.105 (0.071) | 0.697 (0.074) | -0.078 (0.070) | 0.038 (0.067) | -0.109 (0.069) | 0.698 (0.072) |
| numberr | 0.015 (0.070) | -0.110 (0.064) | -0.081 (0.070) | 0.628 (0.073) | 0.015 (0.070) | -0.111 (0.065) | -0.085 (0.068) | 0.634 (0.071) |
| figurer | 0.186 (0.067) | 0.026 (0.059) | -0.018 (0.066) | 0.519 (0.072) | 0.188 (0.065) | 0.027 (0.060) | -0.019 (0.065) | 0.522 (0.071) |
| object | -0.138 (0.068) | -0.049 (0.065) | 0.160 (0.069) | 0.588 (0.077) | -0.136 (0.067) | -0.050 (0.063) | 0.158 (0.068) | 0.589 (0.072) |
| numberf | 0.036 (0.066) | -0.002 (0.060) | 0.078 (0.066) | 0.472 (0.075) | 0.036 (0.065) | -0.003 (0.060) | 0.078 (0.066) | 0.475 (0.074) |
| figurew | 0.030 (0.065) | 0.118 (0.058) | 0.018 (0.065) | 0.367 (0.077) | 0.032 (0.064) | 0.120 (0.058) | 0.017 (0.064) | 0.369 (0.076) |

**Table 6. Comparing standardized factor loading estimates for model 3 BSEM in R vs Mplus: Standardized factor loadings and (posterior standard deviation) for all parameter estimates are reported here.**

# GLOBAL FIT INDICES

| | | Global fit indices | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Number of freely estimated parameters | Chi-square(d.f) (p-value) | RMSEA | CFI | TLI | SRMR | Highest PSR | PPP |
| Model 1 | SAS ML | 44 | 313(146) (<0.0001) | 0.062 | 0.915 | 0.901 | 0.067 | | |
| | Mplus ML estimation | 63 | 314(171) (<0.0001) | 0.062 | 0.915 | 0.900 | 0.064 | | |
| | R ML estimation (lavaan) | 44 | 314(146) (<0.0001) | 0.062 | 0.915 | 0.9 | 0.067 | | |
| Model 2 | SAS ESEM | 89 | 130(101) (0.0267) | 0.031 | 0.985 | 0.975 | 0.024 | | |
| | Mplus ESEM | 101 | 130(101) (0.0251) | 0.031 | 0.985 | 0.975 | 0.023 | | |
| | R ESEM | 101 | 130(101) (0.0251) | 0.031 | 0.985 | 0.975 | 0.024 | | |
| Model 3 | Mplus BSEM estimation | 120 | | 0.033 | 0.982 | 0.972 | | 1.012 | 0.093 |
| | R BSEM estimation | | | 0.032 | 0.982 | 0.972 | | 1.001 | 0.118 |

**Table 7. Comparing global fit indices across software for all 3 models: Available fit statistics are different across software. Model 3 fit statistics RMSEA, CFI, and TLI are Bayesian adjusted fit indices available in R and Mplus. PSR is Gelman Rubin point scale reduction factor. PPP is posterior predictive p-value.**

| | Additional global fit indices | | | |
|---|---|---|---|---|
| | BNFI | BMc | B Gamma Hat | Adjusted B Gamma Hat |
| R BSEM | 0.933 | 0.943 | 0.988 | 0.976 |

**Table 8. Additional Bayesian adjusted global fit indices only available in R.**

| Fit Summary | | |
|---|---|---|
| Modeling Info | Number of Observations | 301 |
| | Number of Variables | 19 |
| | Number of Moments | 190 |
| | Number of Parameters | 89 |
| | Number of Active Constraints | 0 |
| | Baseline Model Function Value | 7.136 |
| | Baseline Model Chi-Square | 2140.786 |
| | Baseline Model Chi-Square DF | 171 |
| | Pr > Baseline Model Chi-Square | <.0001 |
| Absolute Index | Fit Function | 0.4341 |
| | Chi-Square | 130.2347 |
| | Chi-Square DF | 101 |
| | Pr > Chi-Square | 0.0267 |
| | Elliptic Corrected Chi-Square | 126.4255 |
| | Pr > Elliptic Corr. Chi-Square | 0.0442 |
| | Z-Test of Wilson & Hilferty | 1.9322 |
| | Hoelter Critical N | 289 |
| | Root Mean Square Residual (RMR) | 0.0244 |
| | Standardized RMR (SRMR) | 0.0244 |
| | Goodness of Fit Index (GFI) | 0.9569 |
| Parsimony Index | Adjusted GFI (AGFI) | 0.919 |
| | Parsimonious GFI | 0.5652 |
| | RMSEA Estimate | 0.0311 |
| | RMSEA Lower 90% Confidence Limit | 0.0113 |
| | RMSEA Upper 90% Confidence Limit | 0.0455 |
| | Probability of Close Fit | 0.9869 |
| | ECVI Estimate | 1.0698 |
| | ECVI Lower 90% Confidence Limit | 0.9964 |
| | ECVI Upper 90% Confidence Limit | 1.1837 |
| | Akaike Information Criterion | 308.2347 |
| | Bozdogan CAIC | 727.1675 |
| | Schwarz Bayesian Criterion | 638.1675 |
| | McDonald Centrality | 0.9526 |
| Incremental Index | Bentler Comparative Fit Index | 0.9852 |
| | Bentler-Bonett NFI | 0.9392 |
| | Bentler-Bonett Non-normed Index | 0.9749 |
| | Bollen Normed Index Rho1 | 0.897 |
| | Bollen Non-normed Index Delta2 | 0.9857 |
| | James et al. Parsimonious NFI | 0.5547 |

**Table 9. ESEM default global fit indices available in SAS: SAS calculates an extensive list of global fit indices, in addition to providing considerable details regarding estimated models through default options.**

| | Model & Software | Estimation time | |
|---|---|---|---|
| | | Estimation of primary analysis | Estimating null model for additional fit indices |
| Model 1 | SAS ML | 1 second | |
| | Mplus ML | 1 second | |
| | R ML estimation (lavaan) | 1.9 second | |
| Model 2 | SAS ESEM | 1 second | |
| | R ESEM | 5 seconds | |
| | Mplus ESEM | 7 seconds | |
| Model 3 | Mplus BSEM-CL estimation | 9 seconds | |
| | R BSEM-CL estimation (STAN) | 14.5 minutes | 6.5 minutes |

**Table 10. Comparing model estimation times across software: Time estimates using Intel i7-9850H processor with 16 GB of RAM.**

## DISCUSSION

A discussion of standardized factor loadings, global fit indices, and software estimation time is outlined below.

### STANDARDIZED FACTOR LOADINGS

For model 1, only trivial rounding differences are seen across software. Standardized factor loadings and standard errors were similar within 0.002 across software for all items. Given the stability of traditional CFA in software, this is not surprising.

For model 2, SAS provided considerably different standardized factor loading estimates, compared to R and Mplus. SAS provided estimates that were as large as 0.02 standardized factor loading estimates different from the estimates both R and Mplus provided. When comparing R and Mplus, the largest discrepancy they provided was smaller at 0.005. Although SAS did provide larger discrepancies in parameter estimates compared to R and Mplus, the differences were not large enough to lead researchers to different conclusions. Upon further inspection, it was noticed that the EFA standardized parameter estimates in part 1 of the EFA within CFA ESEM approach was the cause of the difference in estimates. This is surprising given the length of time that EFA has been adopted into all software. Degrees of freedom we compared across all software was equal to 101 for each analysis, so there is likely some default setting in SAS that is impacting EFA standardized loading estimates. Further research might consider exploring this issue.

For model 3, R and Mplus provided similar standardized loading estimates across software, with the largest discrepancy for loadings being as small as 0.008. This would not impact conceptual interpretation and is attributed to the difference in non-informative priors in each software as well as the different seed utilized to start each MCMC. The internal seed value is set by each software, so while setting them as the same, it does not guarantee control that the same seed was used.

### GLOBAL FIT INDICES

Evaluation of the global fit indices included comparisons between all three software and estimation models, which are included in Tables 7 through 9. SAS, Mplus, and R provided consistent estimates of chi-squared tests, root mean square error of approximation (RMSEA), the comparative fit index (CFI), the Tucker-Lewis index (TLI), and the standardized root mean squared residual (SRMR) values across ML and ESEM. This analysis confirmed that software estimate degrees of freedom differently, which could be one of the reasons for slightly different measures.

SAS provides an extensive list of global fit indices for SEM modeling, which is well beyond options available in Mplus and R for ML and ESEM approaches. Mplus and R have adopted several Bayesian adjusted fit indices along with the posterior predictive p-value (PPP). The *blavaan* package in R provides additional Bayesian adjusted global fit indices. Currently, the only way to estimate these additional Bayesian adjusted global fit indices is to employ R. Additionally, the information criteria and log-likelihood tests were computed by each software for each of the various models, but were not reported in this study. Given SAS's extensive list of global fit indices, adoption of BSEM methods and Bayesian adjusted fit indices would likely provide a competitive advantage for SAS with their vast global fit options.

### MODEL ESTIMATION TIMES

For all maximum likelihood approaches and ESEM, software choice did not result in substantial differences in time to execute analyses. Loading results into SAS's html browser took longer than the actual analyses. The same was true for ML and ESEM methods in R and Mplus. There was a noticeable difference in estimation times for BSEM in R. Given the free cost of using R,

this could be the only downside to using R for this analysis. SAS could potentially use this as a competitive marketing advantage if BSEM methodology were adopted.

**CONCEPTUAL MODEL INTERPRETATION**

SAS, R, and Mplus all provided evidence for not supporting the traditional CFA measurement model. Even with the slight differences in standardized factor loadings, SAS, R, and Mplus all provided consistent evidence of improved model fit using ESEM when estimating non-primary cross-loadings. R and Mplus provided consistent evidence of improved model fit by using informative minor cross-loadings. The authors had previously estimated BSEM models with no cross-loading and found similar conclusions to poor model fit as was found in Model 1 (Gittner et al., 2022). The use of minor cross-loadings has improved the fit of the model while providing a more realistic relationship among items and factors.

## CONCLUSION

This study highlights the continued support for developing latent variable methodology in software. To gain widespread acceptance of methodology that allows more flexible approaches to the use of model estimation with minor cross-loadings, statistical software needs to be proactive in adopting newer methodology. The improved model fit of ESEM and BSEM methodology are highlighted in this study. Yet the control that BSEM methodology gives to researchers to utilize previous information and specify models is much stronger than ESEM methodology. In practice ESEM could still be considered as an exploratory approach, whereas BSEM is a confirmatory method.

Given the known estimation issues with using Bayesian methods in R and cost of a single Mplus license, SAS stands to uniquely position itself with a competitive advantage by adopting BSEM methodology. The authors hope to see SAS adopting BSEM methodology into the PROC CALIS command or document SEM modeling ability in PROC MCMC command in near future. For now, our suggestion to applied researchers and practitioners is to feel free to use any software package if they are choosing to estimate a model with fixed minor cross-loadings of zero. In the event of a poor model fit, instead of making post hoc modifications, the researchers might consider exploring more flexible approaches that allow more realistic non-zero minor cross-loading estimates to improve model fit. This can be accomplished through ESEM and BSEM techniques. Estimation of ESEM models may be executed in any software, but EFA part 1 results might want to be confirmed for consistency in another software package. To utilize a true confirmatory approach with the BSEM methods, researchers should explore the more user-friendly options in Mplus, over the less user-friendly options in R.

## REFERENCES

Asparouhov, T., & Muthén, B. (2009). "Exploratory structural equation modeling." *Structural Equation Modeling: A Multidisciplinary Journal*, 16(3), 397-438.

Asparouhov, T., & Muthén, B. (2021). "Advances in Bayesian model fit evaluation for structural equation models." *Structural Equation Modeling: A Multidisciplinary Journal*, 28(1), 1-14. http://www.statmodel.com/download/BayesAdvantages18.pdf

Brown, T. A. (2015). *Confirmatory Factor Analysis for Applied Research*. Guilford publications.

Chang, C., Gardiner, J., Houang, R., & Yu, Y. L. (2020). "Comparing multiple statistical software for multiple-indicator, multiple-cause modeling: an application of gender disparity in adult cognitive functioning using MIDUS II dataset." *BMC Medical Research Methodology*, 20(1), 1-14.

Chen, F., Brown, G., & Stokes, M. (2016). "Fitting your favorite mixed models with PROC MCMC." *Proceedings of the SAS Global Forum 2016 Conference*. Cary, NC: SAS Institute, Inc.

De Beer, L.T. & Van Zyl, L.E. (2019). "ESEM code generator for Mplus." Retrieved from https://www.surveyhost.co.za/esem/ doi: 10.6084/m9.figshare.8320250

Gittner, K. B. (2021). "Small sample performance of Bayesian confirmatory factor analysis in the presence of missing data." Doctoral dissertation, University of Northern Colorado. Retrieved from https://digscholarship.unco.edu/dissertations/818/

Gittner, K. B., Ramezani, N., & Mobley, K. (2022, August 6-11). "Embracing cross-loading to Improve Latent Variable Models Fit: A Comparison of Exploratory and Bayesian Structural Equation Modeling." *Joint Statistical Meeting 2022*, Washington DC, United States. https://ww2.amstat.org/meetings/jsm/2022/onlineprogram/AbstractDetails.cfm?abstractid=323486

Gu, F., & Wu, W. (2011). "Using SAS PROC TCALIS for multigroup structural equation modelling with mean structures." *British Journal of Mathematical and Statistical Psychology*, 64(3), 516-537.

Guàrdia-Olmos, J., Peró-Cebollero, M., Benítez-Borrego, S., & Fox, J. (2013). "Using sem library in r software to analyze exploratory structural equation models." *Proceedings of the 59th ISI World Statistics Congress* (pp. 25-30).

Gucciardi, D. F., & Zyphur, M. J. (2016). "Exploratory structural equation modelling and Bayesian estimation." *An Introduction to Intermediate and Advanced Statistical Analyses for Sport and Exercise Scientists*, 172-194.

Gustafsson, J.-E. (2001). "On the hierarchical structure of ability and personality*." Intelligence and personality: Bridging the gap in theory and measurement* (pp. 25–42). Lawrence Erlbaum Associates Publishers.

Guo, J., Marsh, H. W., Parker, P. D., Dicke, T., Lüdtke, O., & Diallo, T. M. (2019). "A systematic evaluation and comparison between exploratory structural equation modeling and Bayesian structural equation modeling." *Structural Equation Modeling: A Multidisciplinary Journal*, 26(4), 529-556.

Harman, H. H. (1976). *Modern Factor Analysis*. University of Chicago press.

Harper, W. V., Eschenbach, T. G., & James, T. R. (2011). "Concerns about maximum likelihood estimation for the three-parameter Weibull distribution: Case study of statistical software." *The American Statistician*, 65:1, 44-54, DOI: 10.1198/tast.2011.09103

Holzinger, K. J., & Swineford, F. (1939). "A study in factor analysis: the stability of a bi-factor solution." *Supplementary Educational Monographs*, 48, xi + 91.

Koizumi, R., & In'nami, Y. (2020). "Structural equation modeling of vocabulary size and depth using conventional and Bayesian methods." *Frontiers in Psychology*, 11, 618.

Liang, X., Yang, Y., & Cao, C. (2020). "The performance of ESEM and BSEM in structural equation models with ordinal indicators." *Structural Equation Modeling: A Multidisciplinary Journal*, 27(6), 874-887.

Ma, H. (2020). "Evaluation of the utility of informative priors in Bayesian structural equation modeling with small samples." Doctoral dissertation, Southern Methodist University. https://scholar.smu.edu/cgi/viewcontent.cgi?article=1004&context=simmons_depl_etds

Marsh, H. W., Muthén, B., Asparouhov, T., Lüdtke, O., Robitzsch, A., Morin, A. J. S., & Trautwein, U. (2009). "Exploratory structural equation modeling, integrating CFA and EFA: Application to students' evaluations of university teaching." *Structural Equation Modeling: A Multidisciplinary Journal*, 16(3), 439–476. doi:10.1080/10705510903008220

Merkle EC, Fitzsimmons E, Uanhoro J, & Goodrich B (2021). "Efficient Bayesian Structural Equation Modeling in Stan." *Journal of Statistical Software*, 100(6), 1–22. doi:10.18637/jss.v100.i06.

Merkle, E., & Jorgensen, T., (April, 2022). "Blavaan incredible slow…speed-up solutions?." *Discussion group for blavaan, an R package for Bayesian latent variable analysis*. https://groups.google.com/g/blavaan/c/jiBuYc1inYE?pli=1

Merkle, E. C., & Rosseel, Y. (2015). "blavaan: Bayesian structural equation models via parameter expansion." *arXiv*, 1511.05604.

Miočević, M., Gonzalez, O., Valente, M. J., & MacKinnon, D. P. (2018). "A tutorial in Bayesian potential outcomes mediation analysis." *Structural Equation Modeling: A Multidisciplinary Journal*, 25(1), 121-136.

Muthén, B., & Asparouhov, T. (2012). "Bayesian structural equation modeling: a more flexible representation of substantive theory." *Psychological Methods*, 17(3), 313.

Muthén, L.K., & Muthén, B.O. (1998-2017). *Mplus User's Guide*. Eighth Edition. Los Angeles, CA.

Osborne, J. W., & Banjanovic, E. S. 2016. *Exploratory Factor Analysis with SAS®*. Cary, NC: SAS Institute Inc.

R Core Team (2022). "R: A language and environment for statistical computing." *R Foundation for Statistical Computing*, Vienna, Austria. URL https://www.R-project.org/.

Raykov, T., & Little, T. D. (1999). "A note on procrustean rotation in exploratory factor analysis: A computer intensive approach to goodness-of-fit evaluation." *Educational and psychological measurement*, 59(1), 47-57.

Reis, D. (2017). "Further insights into the German version of the multidimensional assessment of interoceptive awareness (MAIA)." *European Journal of Psychological Assessment*.

Revelle, M. W. (2015). "Package 'psych'." *The comprehensive R archive network*, 337, 338.

Rosseel Y (2012). "lavaan: An R Package for Structural Equation Modeling." *Journal of Statistical Software*, 48(2), 1–36. doi:10.18637/jss.v048.i02.

SAS Institute Inc. (1999). The Complete Guide to the SAS Output Delivery System, Version 8. Sas Institute.

SAS Institute Inc. (2008). *SAS/STAT® 9.2 User's Guide*. Cary, NC: SAS Institute Inc.

SAS Institute Inc. (2018). *SAS/STAT® 15.1 User's Guide*. Cary, NC: SAS Institute Inc.

Silvestrin M, & T. de Beer L (2022). "esemComp: ESEM-within-CFA syntax composer." R package version 0.2, https://mateuspsi.github.io/esemComp.

Smid, S. C., McNeish, D., Miočević, M., & van de Schoot, R. (2020). "Bayesian versus frequentist estimation for structural equation models in small sample contexts: A systematic review." *Structural Equation Modeling: A Multidisciplinary Journal*, 27(1), 131-161.

Stan Development Team (2022). "RStan: the R interface to Stan." R package version 2.21.7, https://mc-stan.org/.

Xiao, Y., Liu, H., & Hau, K. T. (2019). "A comparison of CFA, ESEM, and BSEM in test structure analysis." *Structural Equation Modeling: A Multidisciplinary Journal*, 26(5), 665-677.

## ACKNOWLEDGMENTS

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