Paper SD-04

Decision-Making using the Analytic Hierarchy Process (AHP) and SAS/IML®

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ABSTRACT

SAS/IML® can be used to implement the Analytic Hierarchy Process (AHP). AHP helps decision-makers choose the best solution from several options and selection criteria. Thomas Saaty developed AHP as a decision-making method in the 1970s. AHP has broad applications in operations research, quality engineering, and design-for-six-sigma (DFSS) situations.

AHP builds a hierarchy (ranking) of decision items using comparisons between each pair of items expressed as a matrix. Paired comparisons produce weighting scores that measure how much importance items and criteria have with each other.

This presentation will model AHP using personal, business, and medical decision-making examples. SAS/IML scripts will generate output that includes measures of criteria and selection importance and data consistency.

INTRODUCTION

This presentation is about one of the most famous methods for making multi-criteria decisions called the Analytic Hierarchy Process or AHP. AHP was developed to optimize decision making when one is faced with a mix of qualitative, quantitative, and sometimes conflicting factors that are taken into consideration. AHP has been very effective in making complicated, often irreversible decisions.

I intend to show how SAS/IML implements AHP to help decision makers choose the best solution among several alternatives across multiple criteria. With SAS/IML, I will build an AHP model using the example of choosing a smart phone. Finally, I will describe some recent uses and provide final notes.

Decision-making involves the use of intelligence, wisdom and creativity in order for humans to satisfy basic needs or to survive. Evaluating a decision requires several considerations such as the benefits derived from making the right decision, the costs, the risks, and losses resulting from the actions (or non-actions) taken if the wrong decision is made.

Decision-making methods range from reliance on chance (such as flipping coins, reading tea leaves or tarot cards) to the use of more structured decision-making tools. Sound decision-making involves weighing all the factors that are important. Six sigma belts must carefully weigh the advantages and disadvantages of decision choices so that the future success and survival in business or life can be optimized, nothing is guaranteed.

Modern Day decision-making has been inherently complex when many factors have to be weighed against competing priorities. One of the modern tools developed in the last 30 years used to assess, prioritize, rank, and evaluate decision choices is the Analytic Hierarchy Process (AHP) developed by Thomas Saaty [1-3, 5-7].

Thomas Saaty developed AHP in the 1970s as a way of dealing with weapons tradeoffs, resource and asset allocation, and decision making when he was a professor at the Wharton School of Business and a consultant with the Arms Control Disarmament Agency. There he was faced with the problem of dealing with high costs and a host of considerations with many factors that conflicted with each other or were not easily specified.

AHP uses the judgments of decision makers to form a decomposition of problems into hierarchies. Problem complexity is represented by the number of levels in the hierarchy which combine with the decision-maker's model of the problem to be solved. The hierarchy is used to derive ratio-scaled measures for decision alternatives and the relative value that alternatives have against organizational goals (customer satisfaction, product/service, financial, human resource, and organizational effectiveness) and project risks. AHP uses matrix algebra to sort out factors to arrive at a mathematically optimal solution. AHP is a time-tested method that has been used in multi-billion dollar decisions.

AHP derives ratio scales from paired comparisons of factors and choice options.

Typical applications where AHP has been used are in:

- Prioritizing factors and requirements that impact software development and productivity,
- Choosing among several strategies for improving safety features in motor vehicles,
- Estimating cost and scheduling options for material requirements planning (MRP),
- Selecting desired software components from several software vendors,
- Evaluating the quality of research or investment proposals.

AHP also uses actual measures like price, counts, or subjective opinions as inputs into a numerical matrix. The outputs include ratio scales and consistency indices derived by computing eigenvalues and eigenvectors.

Saaty allowed some measures of inconsistency (common with subjective human judgment) when applied to the logic of preferences. Inconsistencies arise when comparing three items, A, B, and C. For example, if item A is more preferred over item B, and item B is more preferred over item C, then by the transitive property, Item A should be more preferred over item C. If not, then the comparisons are not consistent.

Measures of inconsistency set AHP apart from other multi-criteria methods like goal programming, Multi-Attribute Utility Theory (MAUT), Conjoint analysis (CA), or Choice experiments. Goal programming applies linear programming to achieve the goals subject to changing objectives constrained by adding slack and other variables representing deviation from the goal.

MAUT [15] assigns numbers that indicate how much attributes are valued by constructing multiattribute utility functions, scaling factors for each attribute, and estimating probabilities of best-case, intermediate-case and worst-case outcomes resulting from the decisions. Adjustment of the attribute scales proceeds until the satisfactory optimal probability is achieved.

AHP is a simpler form of MAUT where the paired comparisons are used to derive the utility functions represented by the priority or weight vector from contributing criteria and alternatives in the hierarchy.

Conjoint Analysis (CA) is a marketing technique used to measure, analyze, and predict how customers are likely to respond to new features or attributes of existing products or new products being developed.

Choice experiments use surveys to query potential customers about important product or service features they might prefer before beginning an expensive development process and waiting for the satisfaction results. Product or service characteristics may change so rapidly that it is crucial to quickly identify the attributes that help the product maker or service provider to design and build prototypes, or pilot test the proposed service(s).

Both CA and Choice experiments use full-, fractional-factorial, or optimal design matrices made up of specified factor-setting combinations. Users rank order the factor-setting combinations or choice sets to determine preferences.

AHP uses derived weights that show the importance of various criteria. CA uses a reverse approach by determining customer "utility curves," that relate attribute-preference features to complex multi-dimensional choices, alternatives, or trade-offs.

CA and Choice experiments do not allow for individual attribute preferences or inconsistency measures, whereas AHP does. To date, no general consensus has been reached that favors one method over the others.

AHP consists of four steps. One, define the problem and state the goal or objective. Two, define the criteria or factors that influence the goal. Structure these factors into levels and sublevels. Three, use paired comparisons of each factor with respect to each other that forms a comparison matrix with calculated weights, ranked eigenvalues, and consistency measures. Four, synthesize the ranks of alternatives until the final choice is made.

Individuals and groups use the AHP preference scale in Table 1 to form the comparison matrices.

TABLE 1: PREFERENCES MADE ON 1-9 SCALE

AHP Scale of Importance for comparison pair (aij)	Numeric Rating	Reciprocal (decimal)
Extreme Importance	9	1/9 (0.111)
Very strong to extremely	8	1/8 (0.125)
Very strong Importance	7	1/7 (0.143)
Strongly to to very strong	6	1/6(0.167)
Strong Importance	5	1/5(0.200)
Moderately to Strong	4	1/4(0.250)
Moderate Importance	3	1/3(0.333)
Equally to Moderately	2	1/2(0.500)
Equal Importance	1	1 (1.000)

The paired comparison scale between the comparison pair (aij) of two items (item i and item j) is as follows:

(item i) 9-8-7-6-5-4-3-2-1-2-3-4-5-6-7-8-9 (item j)

The preference scale for pair-wise comparisons of two items ranges from the maximum value 9 to 1/9 (0.111 in decimal from). Let aij represent the comparison between item-i (left) and item-j (right). If item-i is 5 times (strong importance) more important than item-j for a given criteria or product, then the comparison aji = 1/aij = 1/5 (0.200) or the reciprocal value for the paired comparison between both items.

After the Comparison matrix is formed, AHP terminates by computing an eigenvector (also called a priority vector) that represents the relative ranking of importance (or preference) attached to the criteria or objects being compared. The largest eigenvalue provides a measure of consistency. Consistency is a matrix algebraic property of cardinal transitivity where the equality $a(ij) = 1/a(ji) = a(ji)^{-1}$, and $a(ij) = a(ik)\ a(kj)$ for any index i, j, k. Inconsistencies arise if the transitive property is not satisfied as determined when the largest eigenvalue from the comparison matrix far exceeds the number of items being compared.

EXAMPLE

Our family recently upgraded our smart phones. AHP was applied to help us make our final selection.

Three smart phones (1, 2, 3) were being considered. The four key quality characteristics we had to evaluate were cost, display resolution, battery life, and internal storage.

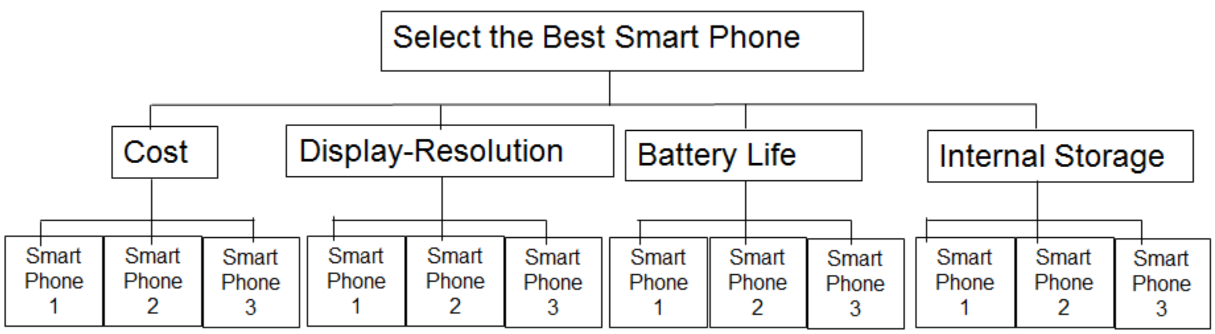
We used AHP to decide which smart phones to purchase based on the attributes desired. Then we weighed and prioritized the alternatives. Our first step in the analytic hierarchy process was to set up the problem. This meant deciding the structure that best represented how the smart phones could be compared over the attributes used for the evaluation. Table 2 summarized the attributes and device choices we considered.

TABLE 2: CRITERIA AND ALTERNATIVES USED IN SELECTING THE SMART PHONES

Attribute	Smart Phone 1	Smart Phone 2	Smart Phone 3
Cost	$549	$450	$550
Display Resolution	950 x 540 pixels	800 x 480 pixels	960 x 640 pixels
Battery Life	7 hrs	7.5 hrs	8 hrs
Internal Storage	16 Gb	16 Gb	16 Gb

Second, we set up the of Parent–Child relationships and produced the Hierarchy of Smart Phone Criteria and Alternative plot displayed in Figure 1.

FIGURE 1 HIERARCHY OF ATTRIBUTES AND DEVICE DECISIONS



The top level of the diagram shows the overall goal of the hierarchy, “Select the Best Smart Phone”. The second level lists the attributes each of the third-tier Smart phones should have.

Third, we set up the paired comparison of each criteria and device choices within each criteria. For each matrix we used the AHPVEC function (Appendix 1) to compute the normalized principal eigenvector that identified the most important factor. Eigenvectors were derived from the eigenvalues of normalized measures (i.e., the proportion of the row/column factors divided the row/column sum). Normalization put the factors on a common scale ranging from 0 to 1. Display 1 showed the paired comparisons of each criteria and phone choices within each criteria we set up.

Display 1: Comparisons Matrices given criteria and preferences

COST

Smart Phone	1	2	3
1	1.00	0.33	1.00
2	3.00	1.00	4.00
3	1.00	0.25	1.00

DISPLAY-RESOLUTION

Smart Phone	1	2	3
1	1.00	2.00	0.50
2	0.50	1.00	0.33
3	2.00	3.00	1.00

BATTERY-LIFE

Smart Phone	1	2	3
1	1.00	3.00	2.00
2	0.33	1.00	0.33
3	0.50	3.00	1.00

INTERNAL STORAGE

Smart Phone	1	2	3
1	1.00	1.00	0.25
2	1.00	1.00	0.25
3	4.00	4.00	1.00

CRITERIA

1.COST 2.DISPLAY-RESOLUTION 3. BATTERY-LIFE 4.INTERNAL-STORAGE

	1.	2.	3.	4
COST	1.00	0.33	0.25	0.50
DISPLAY-RESOLUTION	3.00	1.00	3.00	4.00
BATTERY-LIFE	4.00	0.33	1.00	3.00
INTERNAL STORAGE	2.00	0.25	0.33	1.00

Below is the criteria matrix shown in SAS/IML. The AHPVEC function (Appendix 1) computed the eigenvector weights and consistency measures. The results were output to a SAS/IML matrix which labeled the columns from the rnames and cnames text vectors. The AHPVEC function returned the matrix x5 for further SAS/IML processing.

```
/* call ahpvec function to get ahp for criteria  */

criteria = {1.000 0.333 0.250 0.50,

       3.000 1.000 3.000 4.000,

       4.000 0.333 1 3.000,

       2.0 0.25 0.333 1.000};

x5= ahpvec(criteria);

rnames={"Cost", "Resolution Display", "Battery Life", "Internal Storage"} ;

cnames={"Cost" "Resolution Display" "Battery Life" "Internal Storage" "Priority"
"Geometric Mean" "Lambdamax" "ci" "ri" "cr"} ;

priority_criteria = x5[,5] ; * get priority_criteria vector ;

print x5[rowname=rnames colname=cnames label="AHP Criteria"] ;
```

TABLE 4: PAIRWISE COMPARISON MATRIX FOR THE CRITERIA AND CONSISTENCY METRICS

AHP Criteria										
	Cost	Resolution Display	Battery Life	Internal Storage	Priority	Geometric Mean	Lambdamax	ci	ri	cr
Cost	1	0.333	0.25	0.5	0.0967931	0.0911797	4.2385432	0.0795144	0.9	0.0883493
Resolution Display	3	1	3	4	0.4867755	0.4944647	.	.	.	.
Battery Life	4	0.333	1	3	0.2862346	0.2854079	.	.	.	.
Internal Storage	2	0.25	0.333	1	0.1301968	0.1289476	.	.	.	.

The **Priority** (a.k.a. normalized, principal eigenvector) column is the relative ranking of the criteria produced by dividing each element of the matrix with the sum of its column. Next, the average across the rows is computed. The sum of priority criteria vector is one. The largest value in the priority weight is the most important criterion Display-Resolution= 0.4868.

The **Geometric Mean** is an alternative measure of the **Priority** and was formed by taking the n-th root of the product matrix of row elements divided by the column sum of row geometric means. The **Geometric Mean** agrees closely with the **Priority**.

Lambdamax (4.2385) is an eigenvalue scalar that solved the characteristic equation of the input comparison matrix. Ideally, the **Lambdamax** value should equal the number of factors in the comparison (n=4) for total consistency.

The consistency index (**ci**) measures the degree of logical consistency among pair-wise comparisons. The random index (**ri**) is the average CI value of randomly-generated comparison matrices using Saaty's preference scale (Table 1) sorted by the number of items being considered.

Consistency ratio (**cr**) indicates the amount of allowed inconsistency (0.10 or 10%). Higher numbers mean the comparisons are less consistent. Smaller numbers mean comparisons are more consistent. CRs above 0.1 means the pair-wise comparison should be revisited or revised.

Table 5 combined the vertically-concatenated comparison matrices of the smart phones within each criteria as the synthesized step.

The **Priority** eigenvectors for each criterion were appended into a single, priority-weight matrix. Matrix multiplication of the priority-weight matrix and the criteria-comparison matrix eigenvector produced final_result and Benefit vectors. These vectors were used to form the benefit_cost_ratio matrix of Table 6. The benefit-cost matrix was converted into a SAS dataset using the Create command.

TABLE 5: COMBINED COMPARISON MATRICES OF SMART PHONES WITHIN EACH CRITERIA

AHP of cost									
	Smart Phone 1	Smart Phone 2	Smart Phone 3	Priority	Geometric Mean	Lambdamax	ci	ri	cr
Smart Phone 1	1	0.333333	1	0.1923976	0.1919206	3.0092139	0.0046069	0.58	0.007943
Smart Phone 2	3	1	4	0.6327486	0.633708	.	.	.	.
Smart Phone 3	1	0.25	1	0.1748538	0.1743715	.	.	.	.

AHP Display Resolution									
	Smart Phone 1	Smart Phone 2	Smart Phone 3	Priority	Geometric Mean	Lambdamax	ci	ri	cr
Smart Phone 1	1	2	0.5	0.2972748	0.2969775	3.0088419	0.0044209	0.58	0.0076223
Smart Phone 2	0.5	1	0.333	0.1637311	0.1633785	.	.	.	.
Smart Phone 3	2	3	1	0.5389941	0.539644	.	.	.	.

AHP for Battery									
	Smart Phone 1	Smart Phone 2	Smart Phone 3	Priority	Geometric Mean	Lambdamax	ci	ri	cr
Smart Phone 1	1	3	2	0.5247284	0.5278853	3.0531312	0.0265656	0.58	0.0458028
Smart Phone 2	0.333	1	0.333	0.1414788	0.1395678	.	.	.	.
Smart Phone 3	0.5	3	1	0.3337928	0.3325469	.	.	.	.

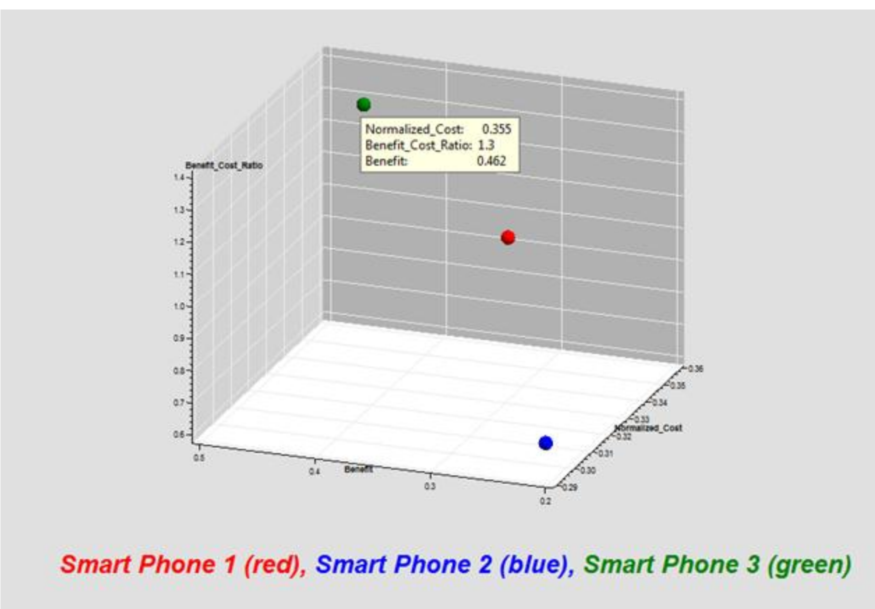
AHP Internal Storage									
	Smart Phone 1	Smart Phone 2	Smart Phone 3	Priority	Geometric Mean	Lambdamax	ci	ri	cr
Smart Phone 1	1	1	0.25	0.1666667	0.1666667	3	0	0.58	0
Smart Phone 2	1	1	0.25	0.1666667	0.1666667	.	.	.	.
Smart Phone 3	4	4	1	0.6666667	0.6666667	.	.	.	.

TABLE 6: FINAL AHP RANKING OF THE BENEFIT-TO-COST RATIO

benefit_cost_ratio				
	Cost	Normalized_Cost	Benefit	Benefit_Cost_Ratio
Smart Phone 1	549	0.3544222	0.3352238	0.9458317
Smart Phone 2	450	0.29051	0.2031416	0.6992585
Smart Phone 3	550	0.3550678	0.4616347	1.3001311

Figure 2 shows a scatterplot using the G3D procedure from the data in Table 6, highlighting Smart Phone 3 as the preferred choice.

FIGURE 2: BENEFIT-COST RATIO SCATTER PLOT



LIMITATIONS AND CONCLUSIONS

Although AHP has been used in many applications of the public and private sectors, Hartwich [4] noted several limitations. One, AHP was criticized for not providing sufficient guidance about structuring the problem to be solved, forming the levels of the hierarchy for criteria and alternatives, and aggregating group opinions when team members are geographically dispersed or are subject to time constraints. Team members may carry out rating items individually or as a group. As the levels of hierarchy increase, so does the difficulty and time it takes to synthesize weights. One remedy in preventing these problems is by conducting “AHP Walk-throughs” (i.e., meetings of decision-making participants who review the basics of the AHP methodology and work through examples so that concepts are thoroughly and easily understood).

Another critique of AHP is the “rank reversal” problem, i.e., changes in the importance ratings whenever criteria or alternatives are added-to or deleted-from the initial set of alternatives compared. Several modifications to AHP have been proposed to cope with this and other related issues. Many of the enhancements involved ways of computing, synthesizing pair-wise comparisons, and/or normalizing the priority/ weighting vectors. These issues have been discussed [4, 11-13].

Other SAS tools that allow analysts to examine other aspects of multiple criteria decision-making that complements AHP include:

- Multicriteria Decision Making (MCDM) methods described by Crissey [20-21];
- Correspondence analysis to establish patterns and relationships between criteria and choices throughout the levels of the hierarchy (from the CORRESP procedure of SAS/STAT®);
- Item Analysis for examining consistency across raters preparing comparison matrices [22];
- Principal components to extract dimensions defined by the criteria and options [23];
- Conjoint Analysis which may be implemented by generating scenarios of product attributes, levels, and possible options with the TRANSREG procedure [24];
- Ordered Weighted Averaging (OWA) using SAS/OR® [25].

Despite AHP’s criticisms, the methodology has been popular, fairly simple to apply, and is the leading approach used for multi-criteria decision-making [14]. Other decision-making alternatives have limitations similar to AHP.

NASA used AHP to design mission-success factors for human MARS Exploration [18].

Ho and Emrouznejad [8] used SAS/OR, AHP, and Goal Programming to evaluate warehouse performance delivering products to customers in a logistic distribution network and provided other AHP examples [9].

AHP was used to extract judgmental forecast adjustments of Myrtle Beach, SC golf-course demand [19].

The AHPVEC function in SAS/IML performs the basic AHP calculations found in most AHP web-based calculators (e.g., www.123AHP.com) or Excel spreadsheets. The AHPVEC function is customizable, flexible, and expandable. Data can be input as one multi-dimensional matrix. Most AHP spreadsheets require users to pick specific sheets of different sizes for paired-comparison data entry. The matrices from the AHPVEC function can be output as SAS datasets via the STORE and CREATE commands in SAS/IML for further data shaping, visualization, and analysis.

This presentation has shown how SAS/IML can be used to implement the AHP methodology. The SAS datasets and output produced are typical of the results found in standard AHP reports. I wrote a similar AHP subroutine that runs on the JMP® scripting language (JSL).

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APPENDIX 1: AHPVEC SAS/IML LISTING

```
proc iml;

start ahpvec(x) ; /* begin module */

x = x ;

rsum = J(ncol(x),1,0) ; * initialize
row vector with zeroes ;

csum = J(1,nrow(x),0) ; *initialize
column vector sums with zeroes ;

*rsum = x[,1] + x[,2] + x[,3] ;

*rowsum = x[,+] ;

*csum = x[1,] + x[2,] + x[3,] ;

*colsum = x[+,] ;

do i = 1 to nrow(x) ;

rsum = rsum + x[,i] ;

end;

do i = 1 to ncol(x) ;

csum = csum + x[i,] ;

end ;

ctotal = csum ;

coltotal = J(ncol(rsum),1,0) ;

do i = 1 to nrow(rsum) ;

coltotal = coltotal + rsum[i,] ;

end;

normatrx = x/ctotal; * compute
normalized matrix ;

weight = normatrx[,+]/nrow(x); *
relative importance weight ;

products = x*weight ;

ratio = products/weight ;

/* concatenate x rowsum, ev, */
```

```
lambdamax = ratio[+,]/nrow(ratio) ;

ci=(lambdamax -
nrow(weight))/(nrow(weight) -1);

* ci is consistancy index measures ;

eigenval = abs(eigval(x)); * use of
eigval funtion to get principal
eigenvalue ;

/* prin_eigenval is the largest */

prin_eigenval = max(eigenval);

/* transpose normalized matrix to
compute column means

and transpose back as priority weight2
column vector

*/

transpose_norms = normatrx` ;

tran_col_mean_norms =
transpose_norms[:,];

weight2 =tran_col_mean_norms` ;

/*compute geometric mean eigenvector */

geom = J(nrow(x),1 ,1) ; * initialize
row vector to 1 ;

do i = 1 to nrow(x) ; * do loop
computes the product of row elements ;

geom = geom#x[,i] ;

end;

geomet_mean = geom##(1/nrow(x)) ; *
take the n-th root of the geometric
products ;

geoprodsum=geomet_mean[+,]; * sum of
the geometric mean row vector ;

geomean_eigenv = geomet_mean/geoprodsum
; * normalized geometric mean
eigenvector ;
```

```
priority = weight;

* ri is the random consistency index ;

if nrow(x) <= 2 then ri = 0;

else if nrow(x) = 3 then ri = 0.58 ;

else if nrow(x) = 4 then ri = 0.90 ;

else if nrow(x) = 5 then ri = 1.12 ;

else if nrow(x) = 6 then ri = 1.24 ;

else if nrow(x) = 7 then ri = 1.32 ;

else if nrow(x) = 8 then ri = 1.41 ;

else if nrow(x) = 9 then ri = 1.45 ;

else if nrow(x) = 10 then ri = 1.49 ;

else if nrow(x) = 11 then ri = 1.51 ;

else if nrow(x) = 12 then ri = 1.48 ;

else if nrow(x) = 13 then ri = 1.56 ;

else if nrow(x) = 14 then ri = 1.57 ;

else if nrow(x) = 15 then ri = 1.59 ;

if ri = 0 then cr = . ;

else cr = ci/ri ; /* create matrices
that will form output SAS datasets */
```

```
x1 = x||priority||geomean_eigenv;

x2=lambdamax||ci||ri||cr;

x3=x2//J(nrow(x)-1,4,.) ;

x5=x1||x3 ;return(x5) ;finish ahpvec;
/* end module */ /* call ahpvec
function to get ahp for criteria */

criteria = {1.000 0.333 0.250
0.50,3.000 1.000 3.000 4.000,4.000
0.333 1 3.000,2.0 0.25 0.333 1.000};

x5= ahpvec(criteria);

rnames={"Cost", "Resolution Display",
"Battery Life", "Internal Storage"} ;

cnames={"Cost" "Resolution Display"
"Battery Life" "Internal Storage"
"Priority" "Geometric Mean" "Lambdamax"
"ci" "ri" "cr"} ;

priority_criteria = x5[,5] ; * get
priority_criteria vector ;

print x5[rowname=rnames colname=cnames
label="AHP Criteria"] ;

/* call ahpvec function to get ahp for
other comparisons */

quit;
```