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Sampling by Reversing The Landmarking Process

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ABSTRACT

Variations of commonly applied landmarking sampling are surveyed. These samplings are “forward” landmarking such that each panel data is created by first selecting landmarks and then including the subsequent observations from the selected landmarks. Unlike these forward landmarking samplings, a “backward” or “reverse” landmarking is proposed with a flexible “progressive” weighting scheme on selecting different competing risks of events and non-events. The backward landmarking sample is compared with those forward landmarking samples on real world mortgage data. Results show that the backward landmarking sample has smaller sampling errors than of those forward landmarking samples.

INTRODUCTION

Landmarking sampling was introduced by van Houwelingen and Putter (2011). It has been widely applied in medical field which interest is often in the probability of the occurrence of an event within a prediction window following a time point (landmark). When all the panel data from the selected landmark time points are stacked up, it becomes so-called supermodel or super dataset. In medical field, the focus is finding the relationship between the event and the biomarkers such that when creating a super dataset, the seasonality is often ignored because the data are not aligned with calendar dates.

In financial data, however, the seasonality must be taken into account when creating samples before conducting analyses. The macro-economic variables are included as the explanatory variables to address the seasonality, and, in some cases, these variables can also explain the big jump such as house foreclosures and other spikes during the financial crisis in 2008. Assuming the data is monthly; therefore, it is ideal to make landmark time points at each month in order to exhaustingly cover the seasonality after each month. To precisely account for the seasonality, furthermore, the monthly event rates in the supermodel must be equal to those in the original data at every time point.

Nevertheless, financial data are often large, and it is costly to create a super dataset from every landmark time point. To reduce the data size, consequently, various samplings are adopted by the financial industry following van Houwelingen and Putter (2011) in order to create a stacked sample with a manageable size.

AN ILLUSTRATION

A super dataset is defined as the compilation of the panel datasets from all landmark time points. The construction is illustrated using a made-up example of 4 loans shown in Table 1. Assuming the censoring date is June 2001, 4 loans are alive in January 2001 with Loan ID 1 experiencing the event of interest (the event, hereafter) in June, Loan ID 2 experiencing the event in April, Loan ID 3 is censored in June, and Loan ID 4 is censored in March due to the occurrence of other events or lost to follow-up. Note that 0/1 in each cell are the values of the event indicator with 1 denoting the occurrence of the event, and 0 otherwise.

Since each observation is created at month-end, the total number in each date can be regarded as the number at risk. Therefore, the hazard rate in each date is the number of events divided by the total number.

When creating all the panel data from each loan beginning at all the landmark dates, the compilation of the super dataset aligned by date is in Table 2. The monthly hazard rates are identical in both tables.

Date	ID 1	ID 2	ID 3	ID 4	Number of Events	Total Number	Hazard
01/2001	0	0	0	0	0	4	0
02/2001	0	0	0	0	0	4	0
03/2001	0	0	0	0	0	4	0
04/2001	0	1	0		1	3	0.3333
05/2001	0		0		0	2	0
06/2001	1		0		1	2	0.5

Table 1 Original dataset

	Stacks of ID 1						Stacks of ID 2				Stacks of ID 3						Stacks of ID 4			Numbr of Events	Total Number	Hazard
Landmark / Date	Jan	Feb	Mar	Apr	May	Jun	Jan	Feb	Mar	Apr	Jan	Feb	Mar	Apr	May	Jun	Jan	Feb	Mar			
1/2001, t=1	0, 1						0, 1				0, 1						0, 1			0	4	0
2/2001, t=2	0, 2	0, 1					0, 2	0, 1			0, 2	0, 1					0, 2	0, 1		0	8	0
3/2001, t=3	0, 3	0, 2	0, 1				0, 3	0, 2	0, 1		0, 3	0, 2	0, 1				0, 3	0, 2	0, 1	0	12	0
4/2001, t=4	0, 4	0, 3	0, 2	0, 1			1, 4	1, 3	1, 2	1, 1	0, 4	0, 3	0, 2	0, 1						4	12	0.3333
5/2001, t=5	0, 5	0, 4	0, 3	0, 2	0, 1						0, 5	0, 4	0, 3	0, 2	0, 1					0	10	0
6/2001, t=6	1, 6	1, 5	1, 4	1, 3	1, 2	1, 1					0, 6	0, 5	0, 4	0, 3	0, 2	0, 1				6	12	0.5

Table 2 Super dataset created from original dataset in Table 1

SAS' SQL has provided an easy way to create a stacked sample. Using the 6 observations of ID 1, for example, first create the table "landmark" with 6 landmarks from January 2001 to June 2001, and, then, a stacked sample can be created with the following code.

```
Proc SQL;
create table stacked_sample as select
a.*,
b.landmark
from ID_1 a, lankmarks b
where a.date>=b.landmark;
quit;
```

Expanding the code by including all IDs, a super dataset of a stacked sample of all panel data from all IDs can be obtained.

APPLICATION

The dataset of harp_historical_data1.txt was downloaded from Freddie Mac's website with monthly performance information of mortgage from June 2012 to June 2018. After cleaning the data, loans beginning with loan age 1 (that is, when they were booked) and with continuous monthly performance records are kept. The total number of loans (identified as ID_Loan in the dataset) is 959,959, and the total number of their monthly observations is 56,119,398. Among these 959,959 loans during the data collection period, there are 4 major events REO (real estate owned, 0.832%), Chargeoff (0.891%), Payoff (41.183%), Others (0.214%), and survival (56.880%). Events are recorded at the end of each loan's lifetime. The numbers of the 4 types of events and survivals by date are in Appendix A.

From the original dataset, a landmarking dataset can be created by first selecting an individual (ID, loan number, etc.) from a time point known as landmark, and, then a panel is formed by selecting all the subsequent observations from the landmark up to the event time or censoring time. This process is called landmarking (van Houwelingen and Putter, 2011). When selecting all the landmarks and stacking up all the landmarking datasets, it becomes the super dataset (super prediction dataset or super stacked dataset by some researchers). Following the illustration in Section II, the super dataset created from the original dataset has the total number of observations 1,898,799,764. Given a landmark and a prediction window, as it has been shown in the illustration table in previous section, numbers of events and survivals of the super dataset are the numbers in the original dataset multiplying *ith* month denoted as t. An example of a partial super dataset with the landmark as of July 2016 is shown in Appendix B. Note that the monthly hazard rates of the super dataset are equal to those in the original data.

Ideally, analyses should be done on the super dataset of all the stacks from all landmarks. However, it is impractical to analyze such a large dataset, and in some cases it is infeasible to create and store a super dataset; therefore, it is desirable to create samples from the super dataset. In the next section, five different samplings are presented. The first 4 are commonly applied in the financial industry while the fifth is proposed by the author.

SAMPLINGS

Five samplings are discussed in this section. From the unstacked original dataset of 56,119,398 observations describe in previous section, the super dataset is created by including all the landmarks and all the observations following the landmarks. Note that the observations at each landmark are survivals only. The super dataset creation is mentioned by Liu (et al., 2019) by allowing the prediction window to vary and not necessarily fall within a fixed time suggested by van Houwelingen and Putter (2011). The prediction window after a landmark is referred as "month after landmark". For convenient purpose, month after landmark begins at 1 at the landmark.

As shown in previous section, the super dataset is large and it is impractical for analyzing the entire data. Similar to how the super dataset is created, therefore, a sample of landmarks can be selected first, and then, all the subsequent observations are selected. Recall that super dataset includes all the landmarks. For identifiability purpose, the data are sorted by loan ID and date before samples of landmarks are selected. Four commonly applied stacked samplings are surveyed. These four samples are created by first selecting a number of landmarks, and, then, all the observations after each landmark are appended to form a panel subsample. The difference among these samplings is how the landmarks are selected.

Sampling 1 Uniformly Spaced Landmarking Sample (uniform sample, hereafter)

Landmarks are selected with an equal distance between any two landmarks. For example, if the first selected landmark is January and the distance is set at 3, then, April, July, October are the selected landmarks. When the distance is 1, then, it becomes the super dataset. Under this sampling, a uniform sample is created by selecting the first time point from each ID as the first landmark, and then the following landmarks are selected with a distance of 6. The uniform sampling is suggested by van Houwelingen and Putter (2011).

Sampling 2 Vertical Random Landmarking Sample (vertical sample, hereafter)

Unlike uniformly spaced sampling, landmarks are randomly selected from all available time points. When all the landmarks are selected, it becomes the super dataset. Under this sampling, a 20% random landmarks are selected to create a vertical sample.

Sampling 3 Horizontal Random Landmarkng Sample (horizontal sample, hereafter)

Landmarks are randomly selected from each prediction time point (month, for example). For example, if a dataset beings in January 2000, then, a set of random landmarks are selected from observations in January 2000, February 2000, and so forth. This sampling can be regarded as a stratified sampling that each month is a stratum. When all the landmarks are selected, it also becomes the super dataset. Under this sampling, a 20% random landmarks are selected from each time point to form a horizontal sample.

Sampling 4 Single Random Landmarking Weighted Sample (single sample, hereafter)

One landmark is selected from each ID (subject, loan, individual, person, etc.). Then, all the following observations are appended to the selected landmark to form a panel subsample. Each observation within the same ID is then weighted by the number of the observations of the ID. Such a weighting is a common practice in regression modeling (Lohr, 2009). Contrary to the super dataset, only one landmark from each ID is selected in this sampling; therefore, it is a subset of the super dataset. Note that this sampling can be regarded as the stratified sampling where observations within each ID is a stratum. Under this sampling, a single sample is created.

Sampling 5 Backward Landmarking Sample (backward sample, hereafter)

The four samplings mentioned above are a subset of the super dataset and the process is forward landmarking meaning the landmarks are selected months forward. Unlike these forward landmarking samples, the backward landmarking sampling is proposed by first selecting events and non-events from each date and, then, landmarks are selected backward from those dates of events and non-events. For example, if there are 100 events and 200 non-events in December 2000, and, if all these loans are selected, then, all the observations associated with these loans prior to December 2000 are selected. In this example, each month before December 2000 including itself is a landmark.

Based on the table in Appendix A (regarded as the size table), the numbers of events vary across time and are much smaller than of the survivals. For illustration purpose, the following progressive weighting in Table 3 deliberately applies more weights on events with relatively less occurrence.

Number of Events or Survivals	% Random Events or Survivals Selected	Weight Assigned
[1, 100]	100	1/1
[101, 500]	90	1/0.9
[501, 1000]	80	1/0.8
[1001, 2000]	70	1/0.7
[2001, 3000]	60	1/0.6
[3001, 4000]	50	1/0.5
[4001, 5000]	40	1/0.4
[5001, 6000]	30	1/0.3
[6001, 7000]	20	1/0.2
7001 and up	10	1/0.1

Table 3 Progressive Weighting

Applying the progressive weights on the events and survivals, the events are over-sampled while the survivals are under-sampled. Note that the weight is the reciprocal of the percentage selected. Obviously, the backward sample is also a subset of the super dataset.

COMPARISONS OF THE FIVE SAMPLINGS

Table 4 shows the number of observations of the super dataset is almost 1.9 billions. It is impractical to analyze such a huge data set. Therefore, the 5 stacked samples are the candidates. Among the 5 stacked samples, the total weight of the single sample and the backward sample is close to the total observations of the super dataset by design.

	# Observations	Total Weight
Original	56,119,398	N/A
Unform	340,315,507	N/A
Vertical	379,716,715	N/A
Horizontal	379,765,804	N/A
Single Sample	28,556,220	1,899,209,658
Backward Sample	295,825,769	1,898,888,910
Super Dataset	1,898,799,764	N/A

Table 4 Numbers/weights of unstacked, four forward samples, and the backward sample

Two landmarks are selected for the comparisons. One is January 2010 and the other is January 2013. The prediction time horizon for both is 24 months. The monthly hazard rate for each event, and the event numbers or weights (for single and backward sample) are in Appendix C.

As it is mentioned previously, the super dataset is the compilation of all the panel subsamples from all the landmarks from the original data. And, all the five samples are the sub-sample of the super dataset. Given a landmark and a prediction time horizon, the sampling errors are measured by MAE (mean absolute error) and RMSE (root mean square error) between a sample and the original dataset.

The curves of the hazards of the events are in Appendix D, and MAEs and RMSEs are provided in Table 5. MAEs and RMSEs are comparable among uniform, vertical and horizontal sample. Single sample has the largest MAE and RMSE while backward sample has the smallest MAE and RMSE as expected.

Event	Sample	January 2010		January 2013	
		MAE	RMSE	MAE	RMSE
REO	Uniform	0.000102	0.00014	0.000029	0.000034
	Vertical	0.000105	0.000141	0.000022	0.000031
	Horizontal	0.000108	0.000149	0.000028	0.000035
	Single	0.000253	0.000334	0.000097	0.000124
	Backward	6.00E-07	6.84E-07	2.78E-06	4.47E-06
Chargeoff	Uniform	0.000082	0.0001	0.000028	0.000035
	Vertical	0.000086	0.000129	0.000028	0.000037
	Horizontal	0.000126	0.000185	0.000031	0.000041
	Single	0.000237	0.000356	0.000083	0.00011
	Backward	7.02E-06	0.00002	3.05E-06	4.18E-06
Payoff	Uniform	0.000067	0.000093	0.000022	0.00003
	Vertical	0.00008	0.000114	0.000021	0.00003
	Horizontal	0.00008	0.000107	0.000019	0.000024
	Single	0.000281	0.000524	0.000064	0.000102
	Backward	1.97E-06	9.68E-06	1.59E-06	3.30E-06
Others	Uniform	0.000445	0.000516	0.000328	0.000428
	Vertical	0.000563	0.000773	0.000189	0.000253
	Horizontal	0.000487	0.000668	0.000161	0.000217
	Single	0.001189	0.001764	0.00051	0.000704
	Backward	0.000147	0.000219	0.000138	0.000201

Table 5 MAE and RMSE of Two Landmarks

CONCLUSION

In the single sample, each individual (ID, loan number, etc.) can be regarded as a stratum and applying a weight equal to the number of the observations within the stratum such that the sum of the all sampling weights is equal to the size of the original data. Because one observation is selected from each individual, every individual of event or non-event has its representation in the sample. However, if an individual (stratum) has 20 observations and the first observation is randomly selected, the sum of sampling weights for the loan is 400 because the panel sample has 20 observations, and each observation has weight 20. If the last observation of the individual is selected, the panel sample has only one observation and the weight 20 is applied to this single observation. Nevertheless, the number of the observations of the individual in the super dataset is 210 ((1+20)x20/2). Therefore, the weight in this example ranges from 20 to 400. The variation is large and that is why the sampling errors of the single sample are the largest.

Among the uniform sample, the vertical sample and the horizontal sample, the uniform sample has the smallest MAE and RMSE in most cases. In the uniform sample, however, the choice of the distance between two landmarks is arbitrary. And, the seasonality effect may be neglected when the distance excludes certain months or quarters. When sampling from the data associated with date, the uniform sample may not be a good choice.

Then, the vertical sampling and the horizontal sampling are the better choice for creating a landmarking sample. Under the horizontal sampling, each time point is a random sample of landmarks being selected; therefore, the seasonality should be well preserved. As for the vertical sampling, it is simple to implement, and it should as well preserve the seasonality. The drawback of the vertical sampling is that the landmarks of a short lifetime individual is less likely to be selected comparing to a long lifetime individual. The drawback of the horizontal sampling is that the landmarks of a long lifetime individual is less likely to be selected comparing to a short lifetime individual.

In this study, the proposed backward sampling has the least sampling errors, and the reason is that the numbers of events are first selected so that they are not underrepresented. The sample size is even less than those of the vertical and horizontal sample. Although the backward sampling requires first creating the size table shown in Appendix A, this extra step is worth it for the small sampling errors.

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CONTACT INFORMATION

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APPENDIX A – NUMBER OF EVENTS BY DATE OF THE UNSTACKED ORIGINAL FREDDIE MAC DATA

Date	REO	Chargeoff	Payoff	Others	Survival	Total	Date	REO	Chargeoff	Payoff	Others	Survival	Total	Date	REO	Chargeoff	Payoff	Others	Survival	Total
Apr-2009	0	0	0	0	154	154	May-2012	67	83	1,874	446	409,497	411,967	Jun-2015	98	127	7,639	105	743,427	751,396
May-2009	0	0	0	0	2,829	2,829	Jun-2012	75	109	2,022	57	441,149	443,412	Jul-2015	110	130	7,247	126	738,007	745,620
Jun-2009	0	0	1	0	9,607	9,608	Jul-2012	67	110	2,338	76	470,410	473,001	Aug-2015	104	114	6,489	102	733,129	739,938
Jul-2009	0	0	3	0	20,438	20,441	Aug-2012	90	129	2,753	79	494,531	497,582	Sep-2015	99	98	6,299	154	728,412	735,062
Aug-2009	0	0	11	1	30,184	30,196	Sep-2012	72	137	2,715	41	519,986	522,951	Oct-2015	108	108	6,000	164	723,826	730,206
Sep-2009	0	0	19	1	36,217	36,237	Oct-2012	74	122	3,299	43	540,723	544,261	Nov-2015	100	67	4,879	185	720,335	725,566
Oct-2009	0	0	27	4	41,093	41,124	Nov-2012	82	142	3,197	40	566,822	570,283	Dec-2015	85	134	5,955	125	715,233	721,532
Nov-2009	0	1	37	4	47,916	47,958	Dec-2012	60	191	3,478	34	587,359	591,122	Jan-2016	108	90	4,132	106	712,180	716,616
Dec-2009	0	1	46	4	56,850	56,901	Jan-2013	84	133	3,483	38	608,204	611,942	Feb-2016	94	107	4,474	316	708,437	713,428
Jan-2010	1	4	42	2	68,060	68,109	Feb-2013	83	143	3,350	34	631,317	634,927	Mar-2016	118	121	6,151	94	703,430	709,914
Feb-2010	1	2	35	1	78,387	78,426	Mar-2013	97	159	3,861	37	654,250	658,404	Apr-2016	98	99	6,572	133	697,947	704,849
Mar-2010	3	4	90	0	86,585	86,682	Apr-2013	103	177	4,365	34	676,907	681,586	May-2016	100	99	7,354	140	691,442	699,135
Apr-2010	5	2	120	3	96,713	96,843	May-2013	75	194	5,047	16	697,160	702,492	Jun-2016	93	110	8,231	77	684,046	692,557
May-2010	3	5	162	1	105,871	106,042	Jun-2013	104	191	5,030	31	715,121	720,477	Jul-2016	94	80	7,605	84	677,214	685,077
Jun-2010	3	3	193	7	112,512	112,718	Jul-2013	124	211	5,409	30	728,877	734,651	Aug-2016	107	109	8,850	124	668,917	678,107
Jul-2010	13	4	217	1	121,638	121,873	Aug-2013	136	172	4,274	23	741,568	746,173	Sep-2016	86	82	8,318	76	661,295	669,857
Aug-2010	12	7	345	7	133,211	133,582	Sep-2013	113	155	3,263	27	752,751	756,309	Oct-2016	95	90	8,229	70	653,623	662,107
Sep-2010	12	9	407	18	146,572	147,018	Oct-2013	138	166	3,235	25	759,076	762,640	Nov-2016	103	83	8,054	69	646,142	654,451
Oct-2010	23	10	494	14	160,961	161,502	Nov-2013	126	123	3,038	24	764,327	767,638	Dec-2016	81	65	7,770	366	638,539	646,821
Nov-2010	10	14	623	12	176,046	176,705	Dec-2013	136	157	3,288	26	768,077	771,684	Jan-2017	100	57	5,602	80	633,532	639,371
Dec-2010	24	17	615	9	193,851	194,516	Jan-2014	136	112	2,308	45	772,572	775,173	Feb-2017	83	54	4,603	68	629,452	634,260
Jan-2011	23	22	429	57	210,607	211,138	Feb-2014	110	102	2,210	53	776,287	778,762	Mar-2017	116	70	5,597	71	624,252	630,106
Feb-2011	17	17	328	26	225,827	226,215	Mar-2014	124	106	3,055	200	777,735	781,220	Apr-2017	75	67	5,195	61	619,470	624,868
Mar-2011	27	19	348	25	237,676	238,095	Apr-2014	102	123	3,593	61	779,037	782,916	May-2017	90	59	6,363	276	613,137	619,925
Apr-2011	32	25	375	29	246,358	246,819	May-2014	102	126	4,033	53	779,190	783,504	Jun-2017	74	52	6,900	61	606,431	613,518
May-2011	37	26	459	34	253,460	254,016	Jun-2014	83	109	4,741	59	778,144	783,136	Jul-2017	79	42	6,320	67	600,265	606,773
Jun-2011	37	38	585	55	259,404	260,119	Jul-2014	76	119	5,345	78	775,803	781,421	Aug-2017	107	49	7,054	139	593,221	600,570
Jul-2011	48	46	610	44	266,808	267,556	Aug-2014	102	113	4,954	76	773,413	778,658	Sep-2017	82	47	5,956	44	587,426	593,555
Aug-2011	76	49	846	78	273,897	274,946	Sep-2014	93	119	4,731	75	771,562	776,580	Oct-2017	78	39	6,414	166	581,037	587,734
Sep-2011	56	49	956	44	282,334	283,439	Oct-2014	119	137	4,917	85	769,508	774,766	Nov-2017	83	49	5,665	108	575,440	581,345
Oct-2011	40	55	1,237	39	292,230	293,601	Nov-2014	85	117	4,201	61	768,229	772,693	Dec-2017	53	45	5,501	190	569,911	575,700
Nov-2011	62	64	1,294	61	302,516	303,997	Dec-2014	141	136	4,936	80	765,616	770,909	Jan-2018	84	35	4,396	67	565,560	570,142
Dec-2011	53	55	1,256	76	313,359	314,799	Jan-2015	120	90	3,847	84	764,257	768,398	Feb-2018	70	36	4,065	68	561,321	565,560
Jan-2012	71	61	1,136	58	325,127	326,453	Feb-2015	102	114	4,555	93	761,985	766,849	Mar-2018	83	36	4,862	370	555,970	561,321
Feb-2012	62	55	1,409	78	339,221	340,825	Mar-2015	110	125	6,743	103	757,569	764,650	Apr-2018	98	26	4,850	54	550,942	555,970
Mar-2012	81	78	1,641	71	361,408	363,279	Apr-2015	102	130	6,555	112	753,651	760,550	May-2018	84	29	5,456	148	545,225	550,942
Apr-2012	52	86	1,661	64	383,483	385,346	May-2015	97	106	6,809	278	749,190	756,480	Jun-2018	72	32	5,327	363	539,431	545,225

APPENDIX B – SUPER DATASET OF THE LANDMARK AS OF JULY 2016

Date	t	Original Unstacked						Super Dataset					
		REO	Charge Off	Payoff	Others	Survival	Total	REO	Charge Off	Payoff	Others	Survival	Total
Jul-2016	1	0	0	0	0	677,214	677,214	0	0	0	0	677,214	677,214
Aug-2016	2	107	109	8,850	124	668,024	677,214	214	218	17,700	248	1,336,048	1,354,428
Sep-2016	3	86	82	8,318	76	659,462	668,024	258	246	24,954	228	1,978,386	2,004,072
Oct-2016	4	95	90	8,229	70	650,978	659,462	380	360	32,916	280	2,603,912	2,637,848
Nov-2016	5	103	83	8,051	69	642,672	650,978	515	415	40,255	345	3,213,360	3,254,890
Dec-2016	6	81	65	7,769	366	634,391	642,672	486	390	46,614	2,196	3,806,346	3,856,032
Jan-2017	7	100	57	5,597	80	628,557	634,391	700	399	39,179	560	4,399,899	4,440,737
Feb-2017	8	83	54	4,596	68	623,756	628,557	664	432	36,768	544	4,990,048	5,028,456
Mar-2017	9	116	70	5,594	71	617,905	623,756	1,044	630	50,346	639	5,561,145	5,613,804
Apr-2017	10	75	67	5,183	61	612,519	617,905	750	670	51,830	610	6,125,190	6,179,050
May-2017	11	90	59	6,356	276	605,738	612,519	990	649	69,916	3,036	6,663,118	6,737,709
Jun-2017	12	74	52	6,884	60	598,668	605,738	888	624	82,608	720	7,184,016	7,268,856
Jul-2017	13	79	42	6,301	66	592,180	598,668	1,027	546	81,913	858	7,698,340	7,782,684
Aug-2017	14	106	49	7,025	137	584,861	592,178	1,484	686	98,350	1,918	8,188,054	8,290,492
Sep-2017	15	82	47	5,933	44	578,755	584,861	1,230	705	88,995	660	8,681,325	8,772,915
Oct-2017	16	76	39	6,390	166	572,084	578,755	1,216	624	102,240	2,656	9,153,344	9,260,080
Nov-2017	17	83	48	5,646	108	566,199	572,084	1,411	816	95,982	1,836	9,625,383	9,725,428
Dec-2017	18	51	45	5,478	189	560,436	566,199	918	810	98,604	3,402	10,087,848	10,191,582
Jan-2018	19	84	34	4,379	66	555,873	560,436	1,596	646	83,201	1,254	10,561,587	10,648,284
Feb-2018	20	70	35	4,041	68	551,659	555,873	1,400	700	80,820	1,360	11,033,180	11,117,460
Mar-2018	21	83	36	4,835	370	546,335	551,659	1,743	756	101,535	7,770	11,473,035	11,584,839
Apr-2018	22	97	26	4,814	52	541,346	546,335	2,134	572	105,908	1,144	11,909,612	12,019,370
May-2018	23	84	28	5,425	146	535,663	541,346	1,932	644	124,775	3,358	12,320,249	12,450,958
Jun-2018	24	68	32	5,287	362	529,914	535,663	1,632	768	126,888	8,688	12,717,936	12,855,912

APPENDIX C – COMPARISONS OF HAZARD RATES FROM 2 LANDMARKS

Landmark Date – January 2010

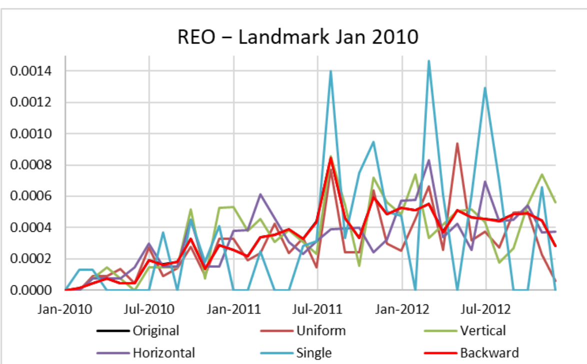
Date	REO						Charge Off						Payoff						Others						Survival					
	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward
Jan-2010	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	68060	22040	13648	13794	68416	68195
Feb-2010	1	0	0	0	9	1	2	1	1	0	0	2	35	9	6	8	27	35	1	0	0	1	0	1	68021	22030	13641	13785	68380	68156
Mar-2010	3	2	1	1	9	3	4	3	1	1	0	4	85	23	13	16	95	85	0	0	0	0	0	0	67929	22002	13626	13767	68276	68064
Apr-2010	5	2	2	1	0	5	2	0	0	0	0	2	106	40	21	16	101	106	3	0	1	0	0	3	67813	21960	13604	13748	68175	67947
May-2010	3	3	1	1	0	3	5	4	1	2	0	5	129	35	23	25	164	134	1	0	0	1	0	1	67675	21918	13577	13721	68011	67804
Jun-2010	3	1	0	2	0	3	3	2	0	0	0	3	155	39	36	30	127	157	7	3	0	4	30	7	67507	21873	13535	13691	67854	67633
Jul-2010	13	6	2	4	0	13	3	1	1	1	0	3	171	46	27	36	190	174	1	0	1	1	0	1	67319	21820	13502	13651	67664	67441
Aug-2010	11	2	2	2	25	11	6	1	1	1	0	6	262	78	46	65	222	261	5	2	1	2	8	5	67035	21737	13451	13582	67409	67158
Sep-2010	12	3	2	2	0	12	8	4	2	2	10	8	293	71	70	65	569	291	15	3	5	3	23	15	66707	21656	13374	13508	66807	66832
Oct-2010	22	6	7	6	30	22	10	5	2	3	0	10	329	90	71	61	339	315	8	2	3	2	0	8	66338	21553	13292	13435	66438	66477
Nov-2010	9	2	1	2	12	9	13	5	2	1	34	13	405	119	67	101	418	417	10	1	3	2	0	10	65901	21426	13220	13328	65974	66027
Dec-2010	19	7	7	2	27	19	13	2	4	1	34	13	369	106	74	80	335	376	6	0	2	2	0	6	65494	21311	13141	13235	65578	65613
Jan-2011	17	7	7	5	0	17	16	6	4	2	17	16	243	71	49	54	313	245	46	20	5	9	73	46	65172	21207	13076	13165	65175	65288
Feb-2011	14	4	5	5	0	14	14	2	4	5	46	14	183	53	30	35	221	185	11	3	3	4	35	11	64950	21145	13032	13118	64873	65064
Mar-2011	22	5	6	8	16	22	14	3	1	0	0	14	173	47	39	32	134	171	9	3	2	1	0	9	64732	21087	12984	13077	64723	64848
Apr-2011	23	9	4	6	0	23	15	3	4	4	0	15	171	53	39	42	174	177	14	6	1	0	38	14	64509	21016	12935	13026	64511	64618
May-2011	25	5	5	4	0	25	16	3	2	3	0	16	186	44	46	33	279	184	12	4	3	2	0	12	64270	20960	12880	12983	64232	64380
Jun-2011	21	7	4	3	18	21	18	7	4	1	0	18	245	68	37	42	152	242	28	9	10	6	41	28	63958	20869	12833	12923	64021	64071
Jul-2011	28	3	3	4	20	28	23	11	5	7	24	23	223	67	42	39	357	232	15	7	3	1	0	15	63669	20781	12779	12873	63620	63772
Aug-2011	54	16	11	5	89	54	25	3	3	1	21	25	350	117	74	71	322	341	23	2	2	4	0	23	63217	20643	12695	12786	63188	63329
Sep-2011	29	5	7	5	21	29	21	9	5	3	53	21	399	115	84	83	315	405	17	5	3	5	23	17	62751	20509	12598	12688	62776	62857
Oct-2011	21	5	2	5	47	21	24	5	4	4	29	24	507	162	101	121	288	511	14	4	3	3	51	14	62185	20333	12485	12558	62361	62287
Nov-2011	37	13	9	3	59	37	29	9	4	7	0	29	565	176	105	120	547	585	14	5	1	4	52	14	61540	20130	12366	12424	61703	61621
Dec-2011	30	6	7	4	31	30	20	5	4	1	26	20	521	157	102	108	756	490	29	11	8	4	56	29	60940	19951	12255	12297	60834	61052
Jan-2012	32	5	6	7	29	32	20	6	5	1	26	20	473	160	94	103	301	470	13	2	2	3	0	13	60402	19778	12150	12181	60478	60517
Feb-2012	31	9	9	7	0	31	22	7	1	4	28	22	523	158	115	106	406	540	28	10	4	6	32	28	59798	19594	12018	12061	60012	59896
Mar-2012	33	13	4	10	88	33	26	6	7	6	0	26	548	187	110	105	663	542	25	6	5	7	32	25	59166	19382	11885	11940	59229	59269
Apr-2012	22	5	5	4	36	22	20	9	4	7	32	20	557	166	98	124	529	555	14	5	3	3	0	14	58553	19197	11773	11804	58632	58657
May-2012	30	18	6	5	0	30	18	7	3	4	34	18	567	169	112	121	554	552	101	36	21	23	257	97	57837	18967	11629	11653	57787	57959
Jun-2012	27	6	6	3	35	27	31	8	10	8	36	33	590	183	108	123	561	603	15	3	2	1	0	15	57174	18767	11509	11512	57155	57280
Jul-2012	26	7	5	8	74	26	29	13	5	3	0	28	669	226	141	129	717	664	20	3	5	1	67	20	56430	18518	11356	11368	56297	56540
Aug-2012	25	5	2	5	39	25	36	12	6	8	104	36	793	261	168	181	882	784	13	6	2	1	0	13	55563	18234	11174	11177	55272	55681
Sep-2012	27	9	3	5	0	27	39	10	7	16	34	37	647	201	106	146	953	681	9	2	0	2	0	9	54841	18012	11045	11021	54285	54925
Oct-2012	27	9	6	6	0	27	27	11	7	6	78	28	841	275	169	163	839	832	9	5	2	1	0	9	53937	17712	10863	10843	53368	54028
Nov-2012	24	4	8	4	35	24	38	15	8	8	71	40	770	262	162	170	673	748	7	4	1	2	0	7	53098	17427	10687	10656	52589	53209
Dec-2012	15	1	6	4	0	15	45	14	4	6	41	40	839	266	154	146	756	874	6	2	2	1	0	6	52193	17144	10522	10498	51792	52274

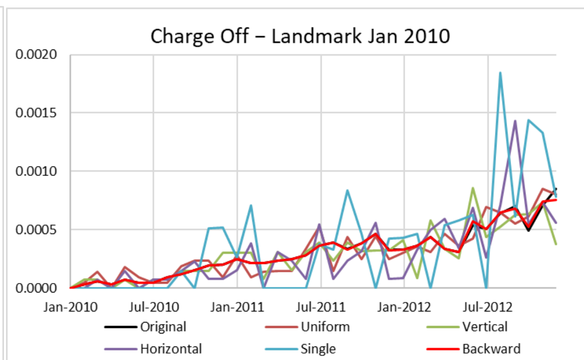
Landmark Date – January 2013

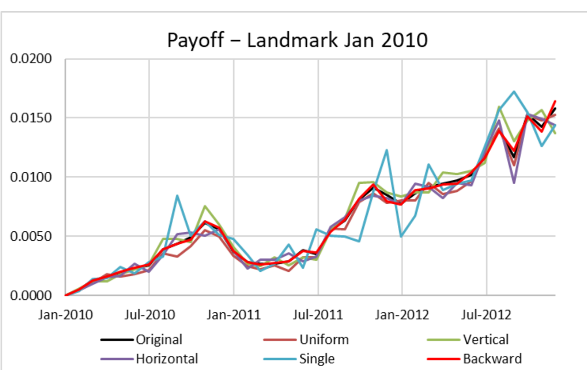
Date	REO						Charge Off						Payoff						Others						Survival					
	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward	Original	Uniform	Vertical	Horizontal	Single	Backward
Jan-2013	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	608204	116879	121685	121475	599403	608007
Feb-2013	83	23	15	13	50	83	143	32	26	18	168	146	3350	635	665	640	3454	3330	34	5	8	5	31	34	604594	116184	120984	120786	595700	604414
Mar-2013	97	13	17	25	140	97	159	28	41	36	154	163	3850	727	757	731	3623	3804	37	8	7	11	5	37	600450	115408	120154	119990	591778	600302
Apr-2013	103	21	20	29	107	102	177	32	35	42	233	177	4337	835	871	860	4020	4417	34	7	8	6	35	34	595799	114513	119206	119067	587383	595571
May-2013	75	18	14	14	79	75	194	39	39	38	202	192	4984	888	1006	996	5101	4946	16	4	4	3	14	16	590530	113564	118145	118014	581987	590341
Jun-2013	104	15	20	26	66	102	191	38	40	31	77	187	4913	934	994	966	4673	4893	29	9	10	8	16	29	585293	112568	117086	116978	577155	585128
Jul-2013	124	22	16	28	74	126	210	34	43	36	210	211	5239	984	1099	1041	5039	5429	27	7	6	6	7	27	579693	111521	115917	115872	571825	579334
Aug-2013	136	28	23	30	134	126	165	31	37	32	155	165	4063	741	826	843	4245	4012	17	4	3	4	19	17	575312	110717	115025	114966	567272	575012
Sep-2013	113	19	21	23	109	113	152	28	30	26	76	150	3051	592	601	591	3423	3068	22	3	5	5	38	22	571974	110075	114370	114319	563626	571659
Oct-2013	136	32	30	29	159	128	159	34	27	32	83	160	2991	539	590	592	2916	2956	20	2	8	6	16	20	568668	109468	113713	113662	560452	568394
Nov-2013	125	26	33	25	11	124	121	17	26	25	141	120	2777	510	561	560	2857	2826	19	6	7	3	0	19	565626	108909	113099	113036	557443	565304
Dec-2013	134	23	30	28	228	130	148	27	40	28	222	148	2970	528	618	566	2944	2962	15	1	4	3	15	15	562359	108330	112422	112396	554034	562048
Jan-2014	130	31	26	28	42	130	102	23	28	23	266	100	2113	410	421	405	1978	2116	39	6	9	9	43	39	559975	107860	111941	111928	551705	559663
Feb-2014	105	14	20	29	80	103	87	16	16	12	79	85	1994	358	417	377	2460	2003	47	9	11	13	89	47	557742	107463	111470	111504	548997	557423
Mar-2014	119	22	24	20	126	121	99	25	21	24	119	98	2708	521	552	533	2775	2706	191	45	42	38	248	186	554625	106850	110836	110884	545729	554311
Apr-2014	92	20	24	16	23	93	109	21	17	18	37	111	3149	585	658	595	2933	3174	51	9	10	8	19	51	551224	106215	110136	110238	542717	550881
May-2014	95	18	20	18	225	95	109	15	18	21	163	113	3487	648	716	697	3441	3512	47	10	5	9	145	47	547486	105524	109372	109498	538743	547113
Jun-2014	73	11	17	10	137	73	93	16	15	18	26	92	4076	762	819	810	3750	4175	51	8	14	11	38	51	543193	104727	108514	108642	534792	542722
Jul-2014	65	6	11	12	37	65	106	20	18	17	56	108	4577	815	888	850	5169	4650	62	13	13	14	26	62	538383	103873	107583	107750	529504	537836
Aug-2014	88	15	18	16	187	90	94	15	17	20	67	94	4151	740	842	852	3948	4292	63	12	14	10	0	63	533987	103091	106695	106849	525302	533296
Sep-2014	75	17	15	17	125	75	95	17	18	16	88	101	4041	761	810	787	4300	4052	62	10	12	10	29	62	529714	102286	105842	106017	520760	529005
Oct-2014	99	14	18	18	249	102	107	23	23	33	64	104	4148	794	857	846	4327	4272	73	18	13	12	131	73	525287	101437	104922	105117	515989	524453
Nov-2014	68	11	20	14	62	68	87	15	18	17	76	85	3573	677	696	714	3943	3552	53	14	7	10	57	53	521506	100720	104185	104358	511851	520694
Dec-2014	114	21	26	26	65	116	103	20	24	16	75	103	4201	800	835	843	3789	4350	62	17	11	9	88	62	517026	99862	103299	103454	507834	516062
Jan-2015	95	17	17	20	135	95	73	11	14	14	71	73	3274	647	654	636	3427	3180	66	15	13	12	50	66	513518	99172	102599	102774	504151	512648
Feb-2015	76	12	18	6	113	73	92	14	18	18	109	91	3962	708	810	773	3860	3950	70	14	13	14	107	70	509318	98424	101751	101952	499962	508463
Mar-2015	93	18	23	17	223	94	91	25	13	17	69	96	5816	1038	1185	1118	5920	5680	73	16	13	18	96	72	503245	97327	100514	100785	493654	502520
Apr-2015	85	19	17	22	62	83	95	22	27	14	37	93	5516	1010	1104	1034	5006	5520	92	20	13	14	205	94	497457	96256	99360	99694	488344	496729
May-2015	69	14	14	17	57	69	81	9	11	16	0	80	5623	1005	1108	1144	4423	5520	229	52	56	48	428	232	491455	95176	98171	98469	483436	490827
Jun-2015	76	12	16	11	111	76	96	24	16	16	36	97	6195	1178	1171	1209	5867	6230	71	15	18	8	0	71	485017	93947	96965	97210	477422	484353
Jul-2015	91	18	17	15	181	92	92	19	20	27	129	91	5750	1035	1175	1153	5673	5950	96	21	24	16	90	97	478988	92854	95732	95996	471349	478121
Aug-2015	77	21	10	14	111	74	82	20	16	15	103	82	5130	944	1035	982	5965	4875	74	16	17	18	37	71	473625	91853	94650	94971	465133	473019
Sep-2015	77	13	15	18	36	77	73	12	12	8	76	73	4956	964	955	988	5335	5225	111	16	22	19	58	110	468408	90848	93650	93934	459628	467534
Oct-2015	74	11	14	11	0	76	75	14	18	15	0	76	4737	881	979	918	4324	4606	131	23	21	28	114	134	463391	89919	92617	92963	455190	462639
Nov-2015	68	9	14	8	0	68	45	10	15	5	148	45	3850	684	809	761	3539	3857	136	31	25	24	148	141	459292	89185	91771	92148	451355	458528
Dec-2015	61	16	6	9	0	61	83	22	17	13	0	82	4659	828	920	956	4715	4573	87	14	17	14	68	85	454402	88305	90815	91152	446572	453725

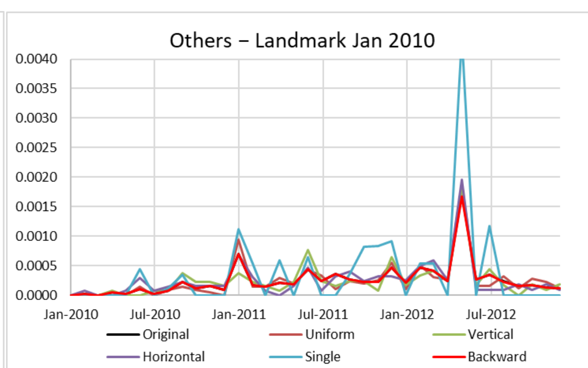
APPENDIX D – FIGURE OF HAZARD CURVES

Landmark Date – January 2010









Landmark Date – January 2013