Two illustrations of the Quantity Theory of Money: A Restatement

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Current draft: July 24, 2023
Link to latest draft

Abstract

Over four decades ago, Robert Lucas established the implications of the Quantity Theory of Money by presenting evidence on the relationship between inflation rates, nominal interest rates, and growth rates of monetary aggregates. Building upon Lucas' work, this study extends the analysis over a longer time span, employs new monetary measurement techniques, and incorporates a long-term interest rate measure. The results of the earlier subsample analysis, utilizing the exponential weighted moving average filter, provide evidence that aligns with Lucas' predictions. Particularly for M1 and M2 money growth measures, the quantity-theoretic predictions are observed, supporting Lucas' propositions. However, beyond the initial period, the conclusions drawn from the Quantity Theory of Money become contingent upon the specific monetary measurement and filtering technique employed.

JEL Codes: E01; E42; E43
Keywords: Monetary Measurement, Inflation Rate, Interest Rate, Filter.

1 Introduction

Lucas (1980) presented empirical evidence to illustrate two central implications of the quantity theory of money in his paper. Firstly, he examined the relationship between changes in the growth rate of money and the inflation rate. Secondly, he investigated the relationship between corresponding and equal changes in the nominal interest rate. Lucas based his analysis on quarterly data spanning 1953:1–1977:4, using M1 as the measure of the money supply, the Consumer Price Index (CPI) as the aggregate measure of prices, and the 90-day Treasury bill rate as the nominal interest rate. To filter the data and focus on long-run relationships associated with the quantity theory, Lucas applied an exponential weighted moving average (EWMA) filter, which reduces the influence of high-frequency fluctuations. His results show that when short-run movements had been judiciously filtered out, M1 growth and CPI inflation and M1 growth and interest rate moved together.

In the current study, two key questions are raised in response to the evidence provided by Lucas to support the two main predictions of the quantity theory. First, are the results unique to his sample? Second, would the results maintain if more care is taken with choices of monetary and interest rate data and a method for filtering the data? To address these questions, I re-examine the conclusions of the original Lucas study with several changes in mind. First, since the time of his investigation, many measurement problems associated with the Federal Reserve's published data have become well-known (Barnett, 1982; Barnett et al., 1992; Barnett, 2011) and superlative indexes of money—whether Divisia or Fisher-Ideal—have been suggested as alternatives that avoid them.

Secondly, considering that the quantity theory focuses on long-run relationships between nominal magnitudes, it is uncertain whether a short-term interest rate like the 90-day T-bill rate is the most appropriate measure to capture the expected inflation effects of changes in the money growth rate. Therefore, I replace the 90-day T-bill rate with the 10-year Treasury note rate in this study.

Finally, at the time of Lucas's writing, using filters to smooth economic time series was not prevalent. One of the first filters to gain wide acceptance and usage is that of Hodrick and Prescott (1997). However, Hamilton (2018) has identified several problems with the Hodrick-Prescott filter and suggested an alternative to it.[1] Recently, Quast and Wolters (2022) propose a simple modification of Hamilton's time series filter that yields smoother

[1] Some of the drawbacks of the HP filter are highlighted by Hamilton (2018) as follows: (a) The HP filter produces series with spurious dynamic relations that have no basis in the underlying data-generating process (b) Filtered values at the end of the sample are very different from those in the middle and are also characterized by spurious dynamics (c) the statistical problem-formalization produces a smoothing parameter whose values are vastly at odds with standard practice (d) there's a better alternative

trend.[2] The current study adopts Lucas' exponential weighted moving average filter and compares the results with the Baxter and King (1999), the Hamilton, and the modified Hamilton filter, investigating its influence on the results.

Also, considering that subsequent studies have identified biases in the CPI, this study utilizes the deflator for personal consumption expenditures (PCE), which measures year-over-year changes in the deflator index. This choice eliminates any potential seasonal effects in quarterly data and captures the aggregate behavior of all agents, which is relevant for evaluating policy responses to inflation.

Over the past few decades, researchers have extensively studied the long-run relationship between money growth and inflation. Some studies based on cross-sectional data have consistently shown a strong positive correlation between money growth and inflation. Notable examples include Dwyer and Hafer (1988), Barro (1997), Pakko (1994), Rolnick and Weber (1994), and Mccandless and Weber (1995), all of which provide empirical evidence supporting this relationship.

However, some studies present conflicting empirical evidence, challenging the notion of a high correlation between money growth and inflation. For instance, Sargent (1982) observed that inflation rates decreased more than money growth rates following monetary reforms in European countries during the 1920s. Similarly, Smith (1985) found that prices did not rise at the same pace as money during the colonial period in the United States. In the 1980s, Friedman (1988) identified a breakdown of the one-to-one relationship between money and prices. Moreover, Wang (2017) reevaluated U.S. data and found weak support for Lucas's results, indicating that the quantity theory of money only holds well for a specific period from 1953 to 1977. Beyond that period, the relationship begins to deteriorate.

I examine the data in Section 2. To illustrate the importance of money measurement, I plot a graph comparing M1, M2, and MZM growth measures using both Divisia and simple-sum methods. This analysis highlights the relevance of accurate money measurement.

In Section 3, I investigate the relationship proposed by Lucas through several graphs. First, I analyze the entire data sample with and without applying data filtering. I use different filters to compare the trend component relationship among the variables of interest. Additionally, I explore the relationship between money and inflation and money and the interest rate for different periods within the data.

The results of the earlier subsample analysis, employing the exponential weighted moving average filter, align with Lucas's predictions. Specifically, for M1 and M2 money growth measures, the evidence supports the quantity-theoretic propositions advocated by Lucas.

[2] The original Halmiton filter relies on 8 quarter ahead forecast errors, whereas the modified Hamilton filter is based on the mean of 4 to 12 quarter ahead forecast errors.

However, beyond the initial period, the conclusions derived from the Quantity Theory of Money depend on the specific monetary measurement and filtering technique used. The choice of monetary measurement and filtering method significantly influences the relationship between inflation rates, nominal interest rates, and the growth rates of monetary aggregates.

2 Data and Data Processing Methods

I analyze the relationships between different measures of the money supply and two key variables: the GDP Deflator, which serves as an indicator of the overall price level, and the 10-Year Treasury Note, which represents the interest rate. For comparison, Appendix (A) presents graphical illustrations using the short-term interest rate. Specifically, I focus on the Federal Reserve's official measures of M1 and M2 aggregates for the money supply. M2 includes M1, savings deposits (including money market deposit accounts), small-denomination time deposits (amounts less than $100,000), and balances in retail money market mutual funds. To address concerns about the accuracy of simple-sum aggregates, which are susceptible to the Barnett critique (Chrystal and MacDonald, 1994), I also examine the Divisia counterparts provided by the Center for Financial Stability.

Following the practice in some recent literature on the measurement of money,[3] I consider the "money, zero maturity" (MZM) aggregate, which was initially proposed by Motley (1988). MZM is computed by the Federal Reserve Bank of St. Louis as M2 minus small-denomination time deposits plus institutional money funds. I use the simple-sum MZM and Divisia version of MZM for this study as described by Barnett et al. (2013).

All the data used in this study are quarterly, matching the frequency of the original Lucas study and the availability of the GDP deflator series. The dataset covers the period from 1968:1 onwards, primarily due to the availability of the Divisia monetary aggregates. It extends until 2019:4, enabling an update of the Lucas (1980) study and testing the validity of his hypotheses beyond the original scope. Data on inflation and interest rates are obtained from the Federal Reserve Economic Data (FRED) repository.

In Figure 1, I plot the differences between year-to-year growth rates for the Divisia and simple-sum M1, M2, and MZM measures. This graph visually demonstrates the empirical significance of measurement inferences and reinforces the findings of Belongia (1996), Hendrickson (2014), and Belongia and Ireland (2016), emphasizing the importance of "measurement matters" in empirical research in monetary economics.

To compute growth rates for the variables, I employ the following transformation:[4]

[3]See Šustek (2010) and Belongia and Ireland (2016).

[4]For Equation (1) and (2), it is worth noting that the transformed variables in these equations offer a

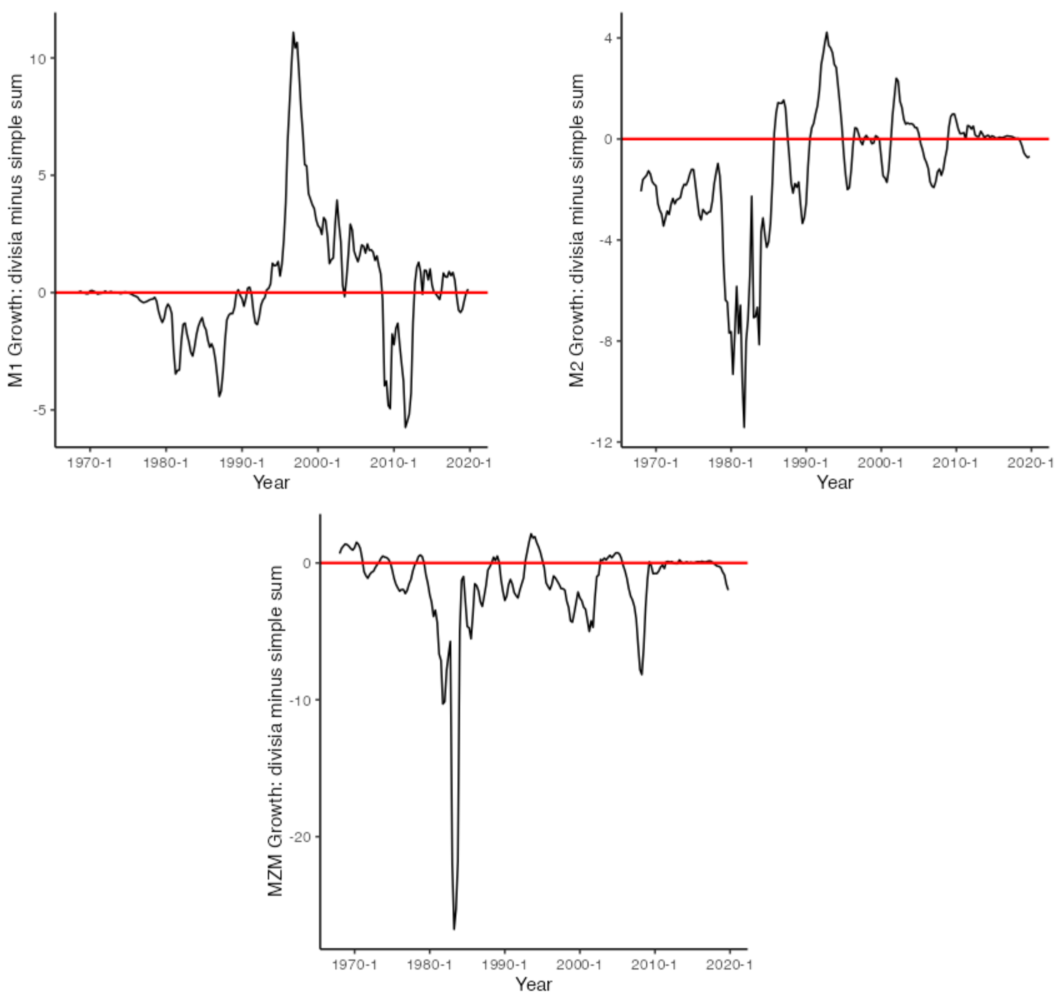
$$X_{0t}^{i} = \left(\frac{M_t - M_{t-4}}{M_{t-4}} \right) \times 100 \tag{1}$$

$$X_{1t} = \left(\frac{P_t - P_{t-4}}{P_{t-4}} \right) \times 100 \tag{2}$$

$$X_{2t}^{j} = r_t \tag{3}$$

where i = simple-sum aggregate M1, M2, MZM, Divisia M1, Divisia M2 and Divisia MZM ; and j = 10–Year Treasury Note. Equation (1) and (2) is the year-to-year growth rates of money and price respectively.

Figure 1: Differences in Year-Over-Year Growth Rates of Divisia and Simple-Sum Monetary Aggregates in Percentage Points.



linear approximation of the natural logarithmic differences used by Lucas (1980). I opted for this approach to avoid any mathematical difficulties that may arise when applying natural logarithms to negative values.

To smooth the original series in my study, I employ various filtering techniques, including the exponential weighted moving average (used by Lucas in his 1980 paper), the Baxter and King (1999) filter, the Hamilton (2018) filter, and the modified Hamilton filter by Quast and Wolters (2022). Lucas varied the smoothing parameter values in his paper to explore the impact of different levels of smoothing on the relationship between money growth, inflation, and interest rates. The smoothing parameter determines how much weight is given to past observations when calculating the filtered series, allowing for control over the degree of smoothing applied to the data. This investigation helps understand how different levels of smoothing affect the observed patterns and correlations.

In the current study, I vary the max-period and min-period window range in the Baxter and King (1999) filter for several reasons. Firstly, adjusting the window range enables capturing different frequencies of fluctuations in the data. A wider window range, covering a longer period, captures slower-moving or long-term trends, while a narrower window range focuses on shorter-term variations. This variation helps gain a more comprehensive understanding of the underlying patterns by exploring the data at different frequencies.

Secondly, the choice of window range affects the level of noise or volatility present in the filtered data. A wider window range, including more historical observations, leads to a smoother and less volatile filtered series. Conversely, a narrower window range captures more short-term fluctuations, resulting in higher volatility in the filtered series.

Lastly, varying the window range provides insights into the stability and robustness of the relationships being studied. Different window ranges may yield different patterns and correlations, indicating the sensitivity of the results to the chosen window size. This analysis allows for assessing whether the relationships hold consistently across different time spans or if they are specific to certain periods. By varying the window range, I can gain valuable insights into the behavior of the variables under investigation and the reliability of the observed relationships.

3 Illustration

3.1 The Full Sample: Original Data for 1968 – 2019

3.1.1 Original Data without Filter

To investigate the relationships proposed by Lucas, I initially analyzed the original full sample data without any filtering. Scatter diagrams are utilized to depict the relationships between inflation and interest rates against different measures of the money supply. The left side displays the simple-sum aggregates, while the right side shows the corresponding Divisia aggregates. Figures 2 and 3 reveal that regardless of the measurement used for the money supply, there is no apparent relationship between money growth and inflation or interest rates. The regression coefficients in the unfiltered data column of Table 1 support these graphical findings. Specifically, the simple-sum and Divisia aggregates indicate a decreasing linear relationship between money growth and inflation. However, only the Divisia aggregates display coefficients suggesting a decreasing relationship between money growth and interest rates. Conversely, the coefficients for the simple-sum aggregates exhibit mixed signs and are mostly statistically insignificant.

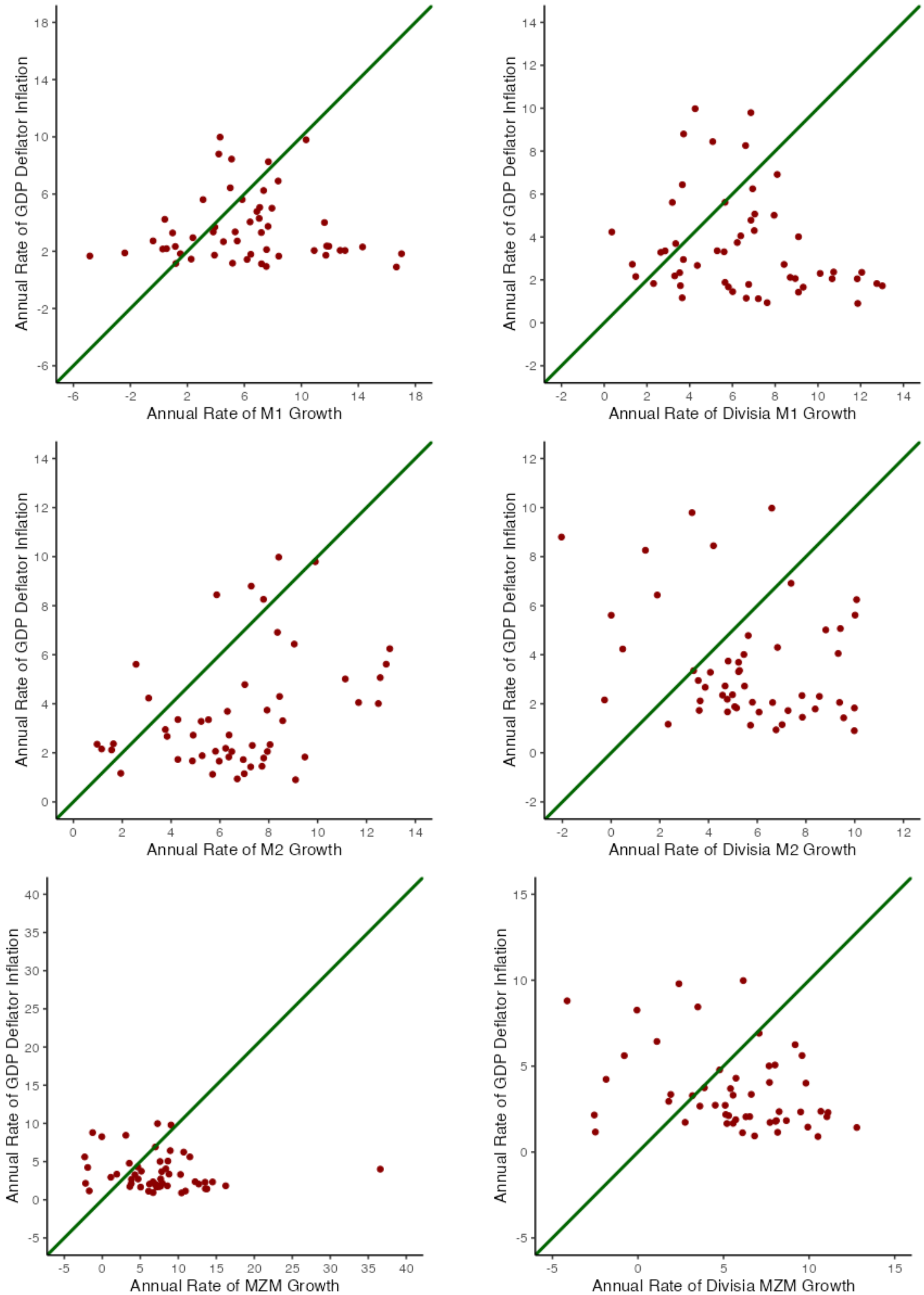
These results, indicating the absence of a significant relationship between money growth and inflation or money growth and interest rates, can be interpreted as reflecting the implementation of counter-cyclical policy measures. Central banks can utilize monetary policy tools, such as adjusting interest rates and reserve requirements, to control the money supply and counteract inflation. Additionally, by actively managing interest rates, central banks can influence borrowing costs, investment levels, and aggregate demand, thereby stabilizing the overall economy. The findings underscore the effectiveness of counter-cyclical policies in mitigating inflationary pressures and independently influencing interest rates to achieve desired macroeconomic outcomes.

3.1.2 Smoothed Data

Figures 4 to 7 illustrates the plots of exponential weighted moving average (EWMA) used by Lucas (1980). In Figures 4 and 5, it is evident that a filter with smoothing parameter $\beta = 0.5$ does not quite produce the quantity-theoretic prediction. However, as the smoothing parameter becomes larger, $\beta = 0.95$, as depicted in Figures 6 and 7, the scatter plots align more closely with the 45° line. Moreover, the scatter plots for the simple-sum aggregates of money growth and interest rates exhibit better alignment compared to the Divisia counterpart. The regression coefficients in the EWMA filter column of Table (1) further support these observations, revealing a strong positive correlation between money growth and inflation as well as money growth and interest rates, particularly for the simple-sum monetary measurement.

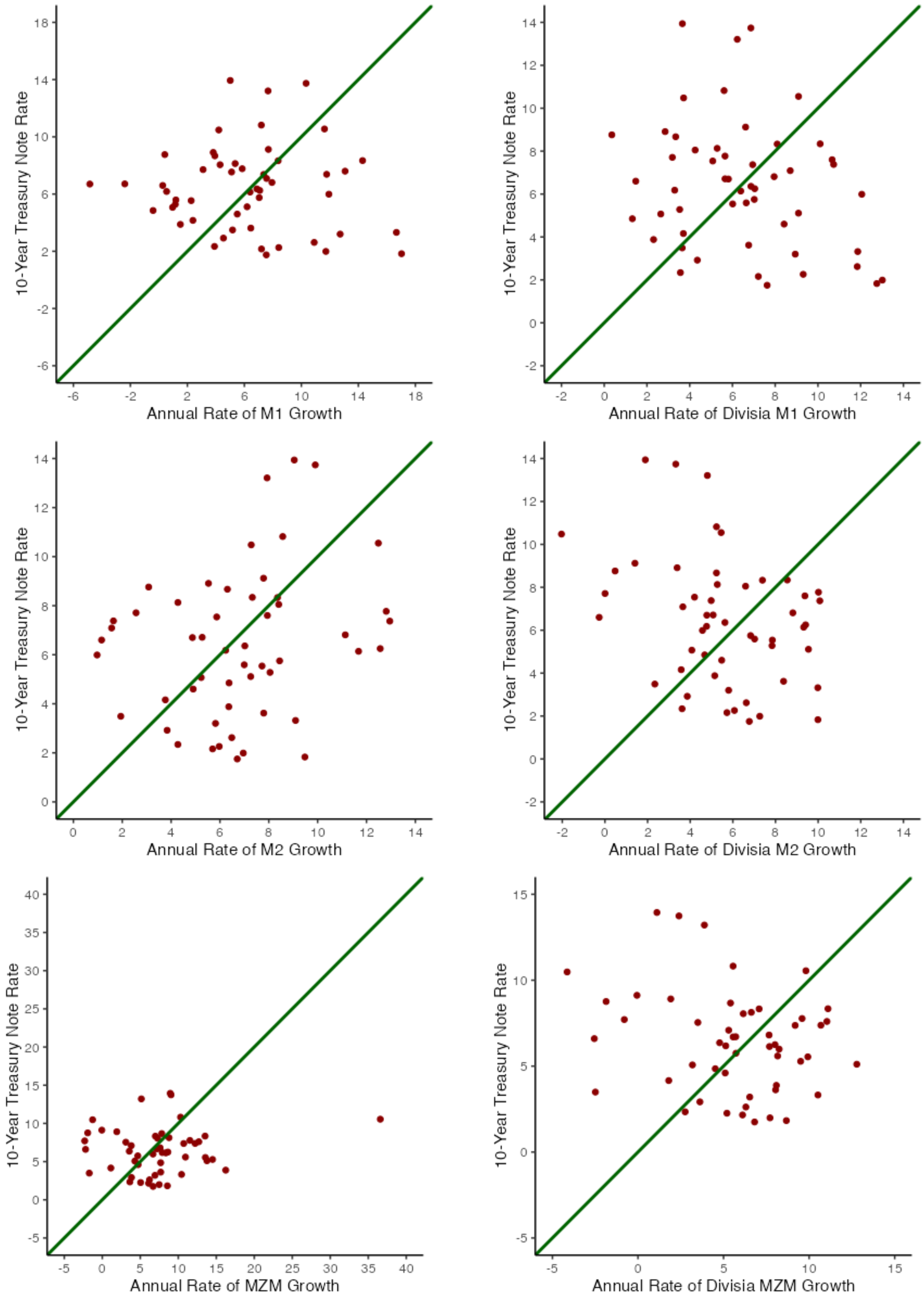
Figures 8 to 13 present the plots generated using the BK filter under different window ranges. Notably, as the window range widens, the scatter plots approach the 45° line, especially for the simple-sum aggregates. The regression coefficients in the BK filter column of Table 1 reinforce this pattern by indicating a strong positive relationship between money growth and inflation, as well as money growth and interest rates. Moving on to the Hamilton filter results shown in Figures 14 and 15, and the modified Hamilton filter in Figures 16 and 17, it is observed that the scatter plots derived from the Divisia aggregates perform

Figure 2: Scatter Plots of Unfiltered Inflation and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 3: Scatter Plots of Unfiltered Long-term Interest Rate and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

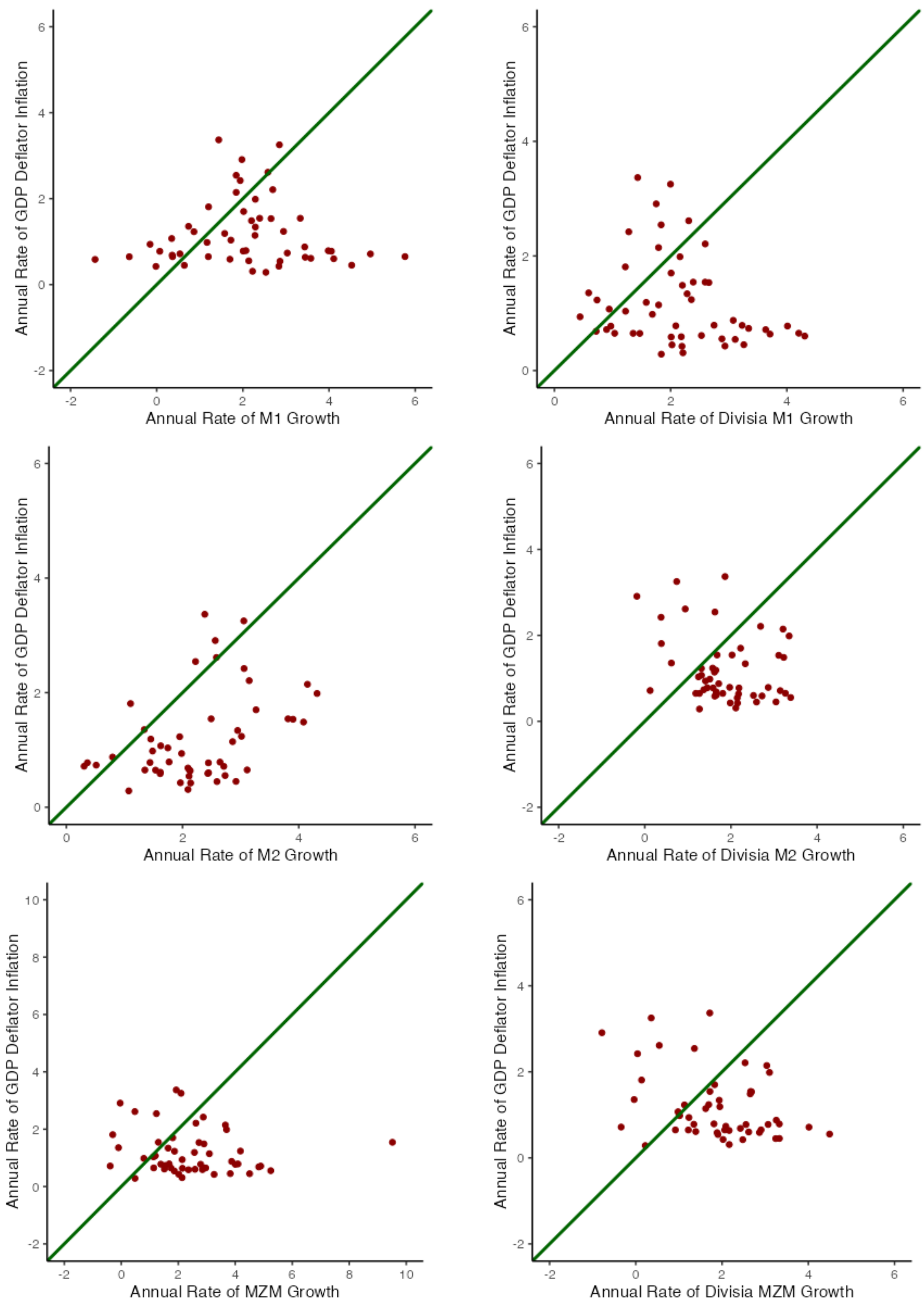
better in conforming to the quantity-theoretic predictions. The corresponding regression coefficients in Table 1 also validate this observation, demonstrating that the quantity-theoretic predictions are generally more robust for the Divisia money aggregates than their simple-sum counterparts.

Table 1: Coefficients of the Regressions on Unfiltered and Filtered Data, 1968–2019

Variable	Unfiltered Data		EWMA Filter		BK Filter		Hamilton Filter		Modified Hamilton Filter	
	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate
M1	–0.01	–0.04	0.22***	0.33***	0.10**	0.04	–0.03	–0.16	0.05	–0.07
	(0.04)	(0.05)	(0.05)	(0.07)	(0.05)	(0.07)	(0.10)	(0.15)	(0.11)	(0.17)
M2	–0.28***	0.25***	0.71***	0.79***	0.69*	0.48***	0.64***	0.63***	1.01***	0.93***
	(0.05)	(0.07)	(0.04)	(0.08)	(0.05)	(0.09)	(0.10)	(0.17)	(0.09)	(0.17)
MZM	–0.07**	0.06	0.11**	0.60**	–0.01	0.39***	0.31***	–0.19	0.33***	0.001
	(0.03)	(0.04)	(0.05)	(0.06)	(0.06)	(0.08)	(0.10)	(0.16)	(0.13)	(0.21)
Divisia M1	–0.18***	–0.25***	–0.02	0.29***	–0.30***	–0.55***	0.32	0.91***	0.06	0.65*
	(0.05)	(0.07)	(0.067)	(0.11)	(0.10)	(0.12)	(0.26)	(0.40)	(0.25)	(0.39)
Divisia M2	–0.22***	–0.38***	–0.01	–0.005	–0.19*	–0.86***	0.59*	2.02***	–0.10	1.12**
	(0.06)	(0.07)	(0.09)	(0.14)	(0.10)	(0.12)	(0.35)	(0.52)	(0.30)	(0.47)
Divisia MZM	–0.23***	–0.22***	–0.22***	0.16	–0.53***	–0.68***	0.54***	0.83***	0.37***	0.83***
	(0.04)	(0.05)	(0.08)	(0.11)	(0.09)	(0.12)	(0.11)	(0.17)	(0.12)	(0.19)

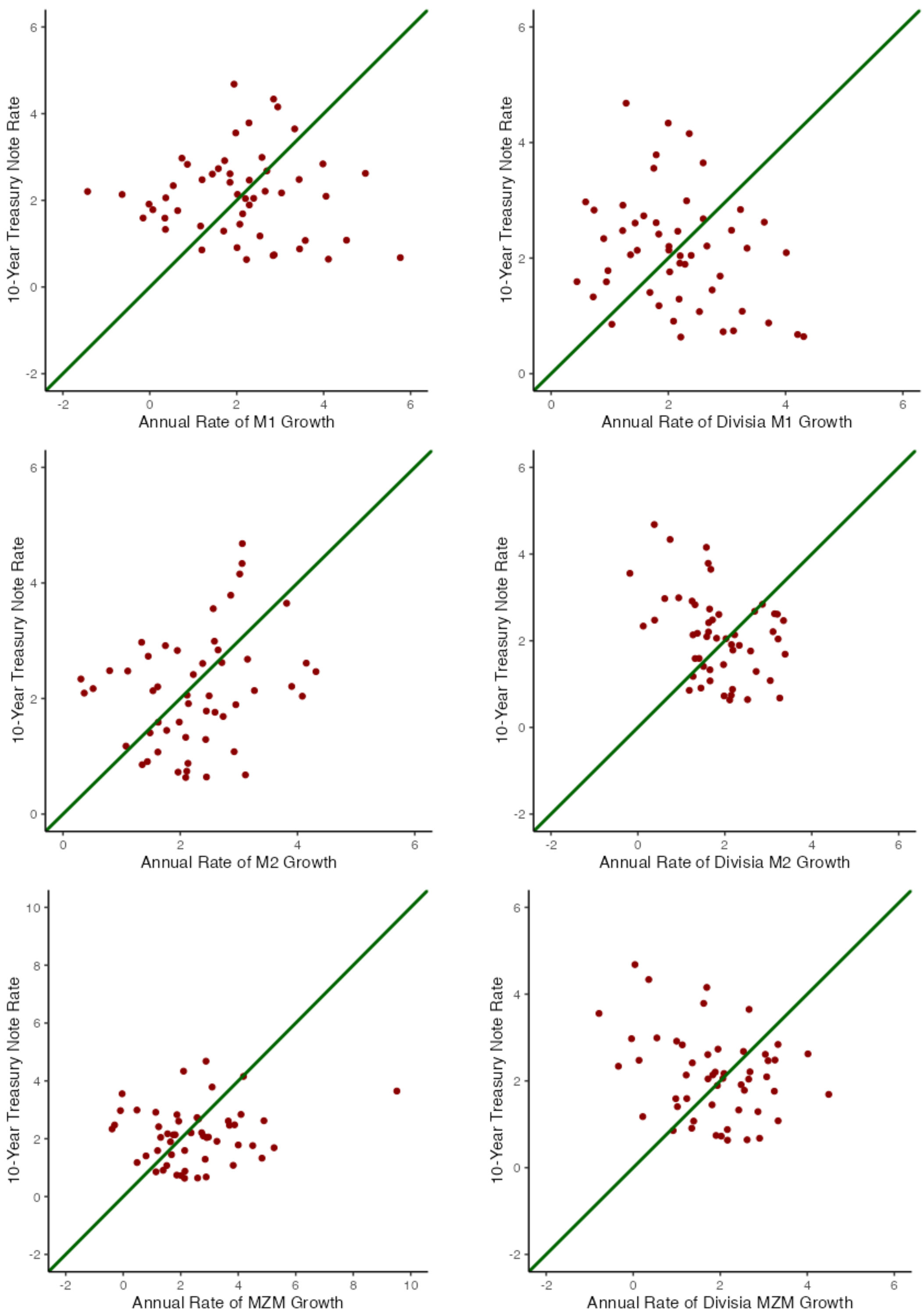
Notes: *** p<0.01, **p<0.05, *p<0.10. Standard errors in parenthesis. Inflation counts for GDP deflator; Interest rate counts for 10–Year Treasury Note

Figure 4: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.5$); 1968–2019



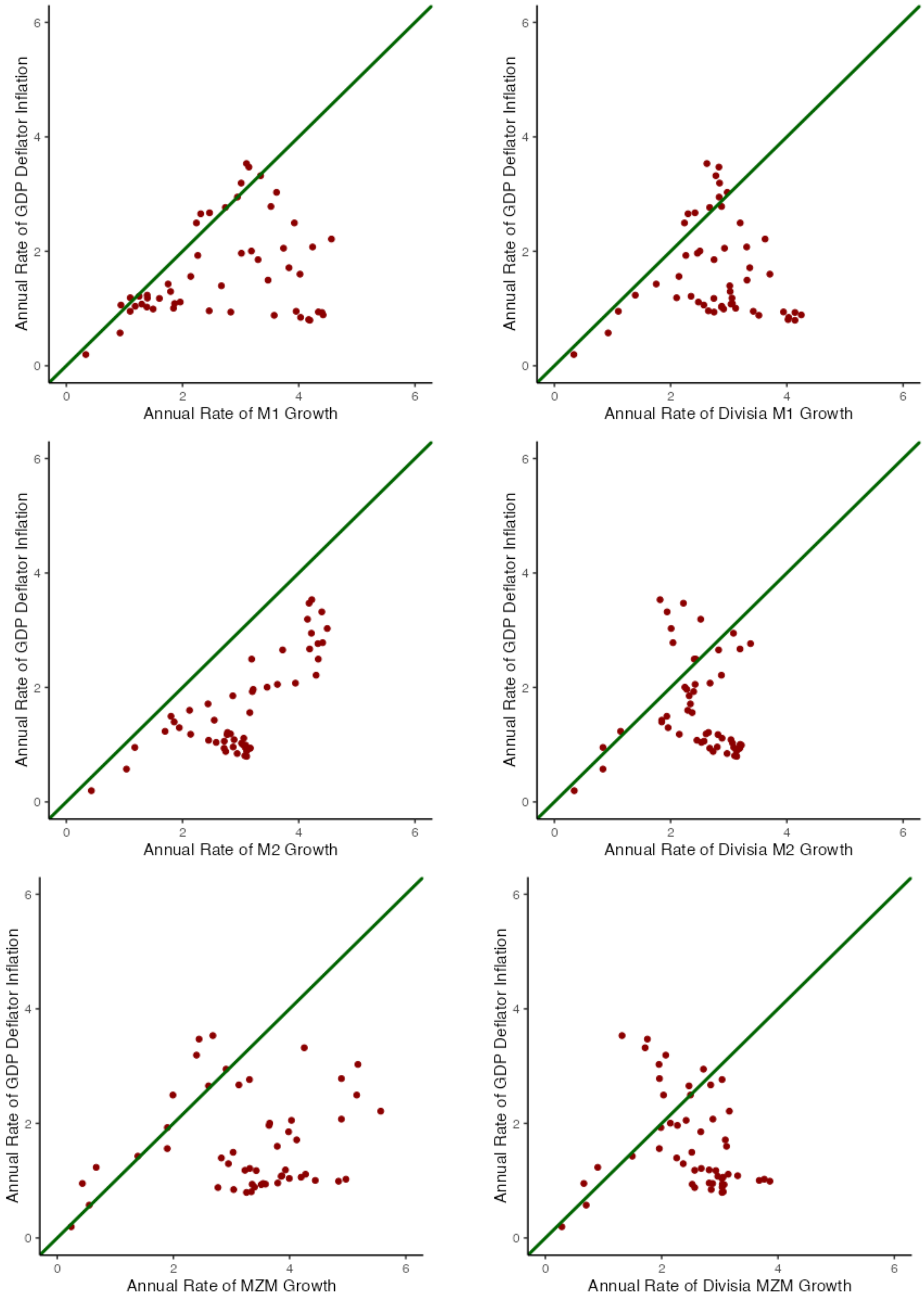
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 5: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.5$); 1968–2019



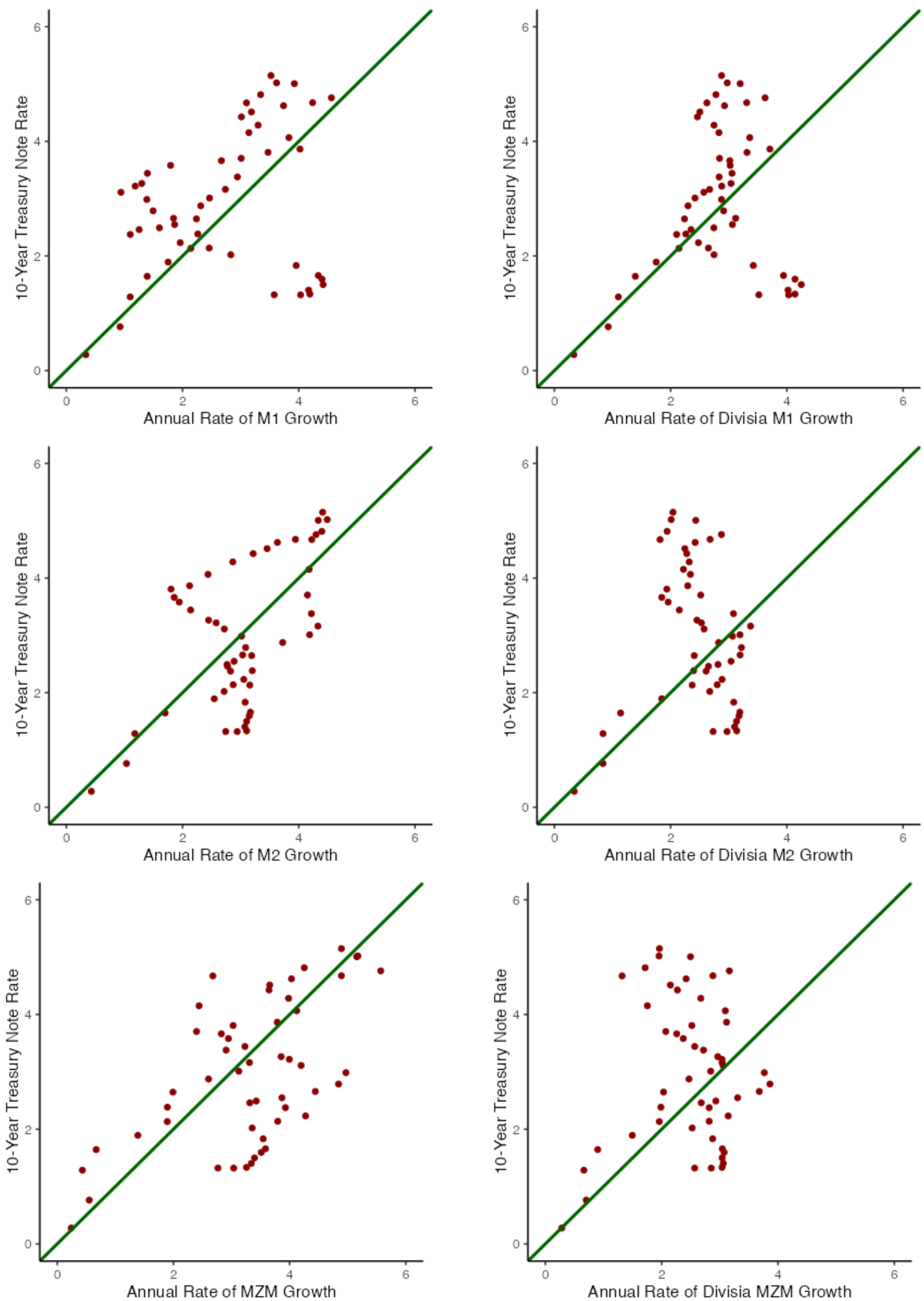
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 6: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.95$); 1968–2019



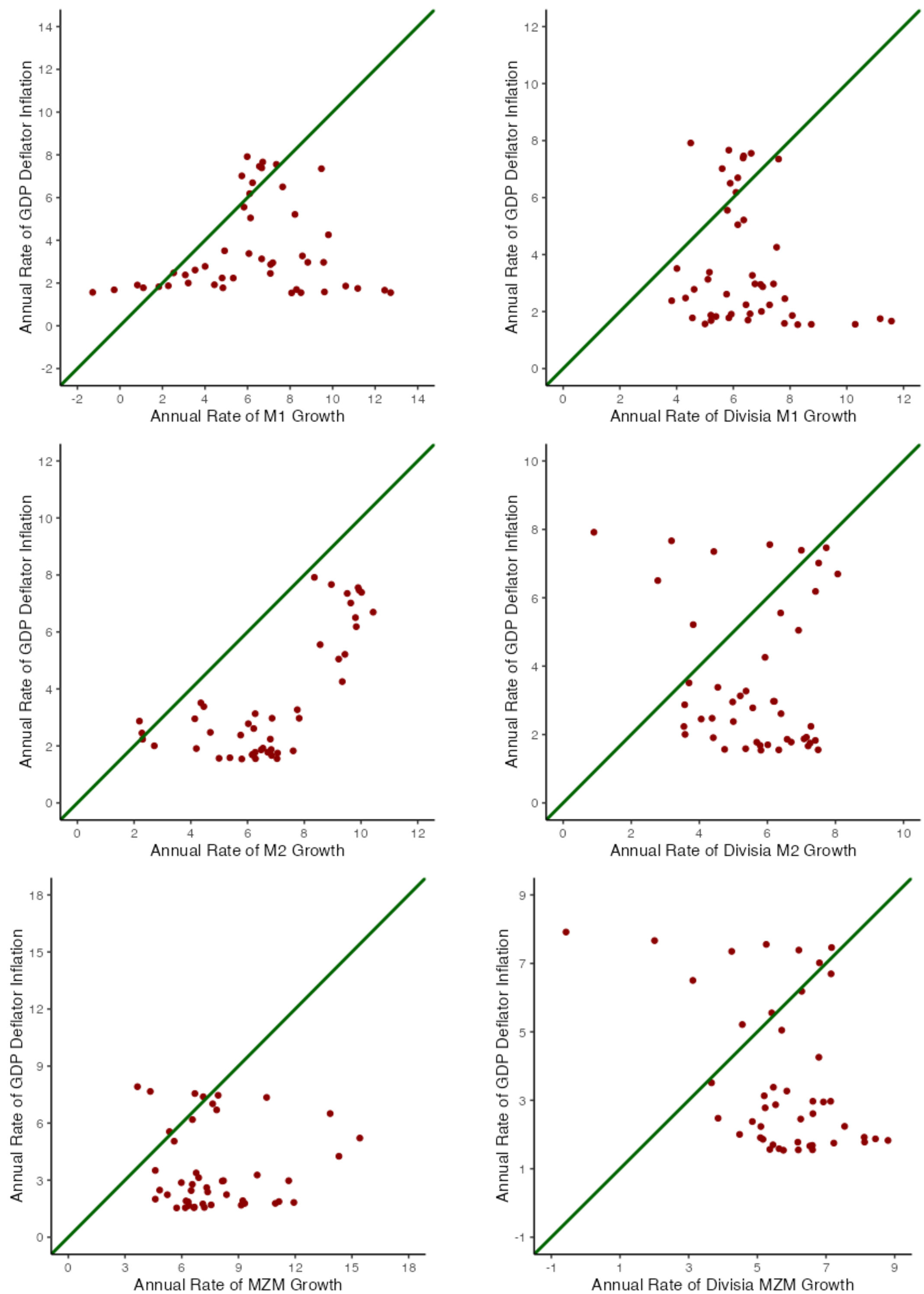
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 7: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.95$); 1968–2019



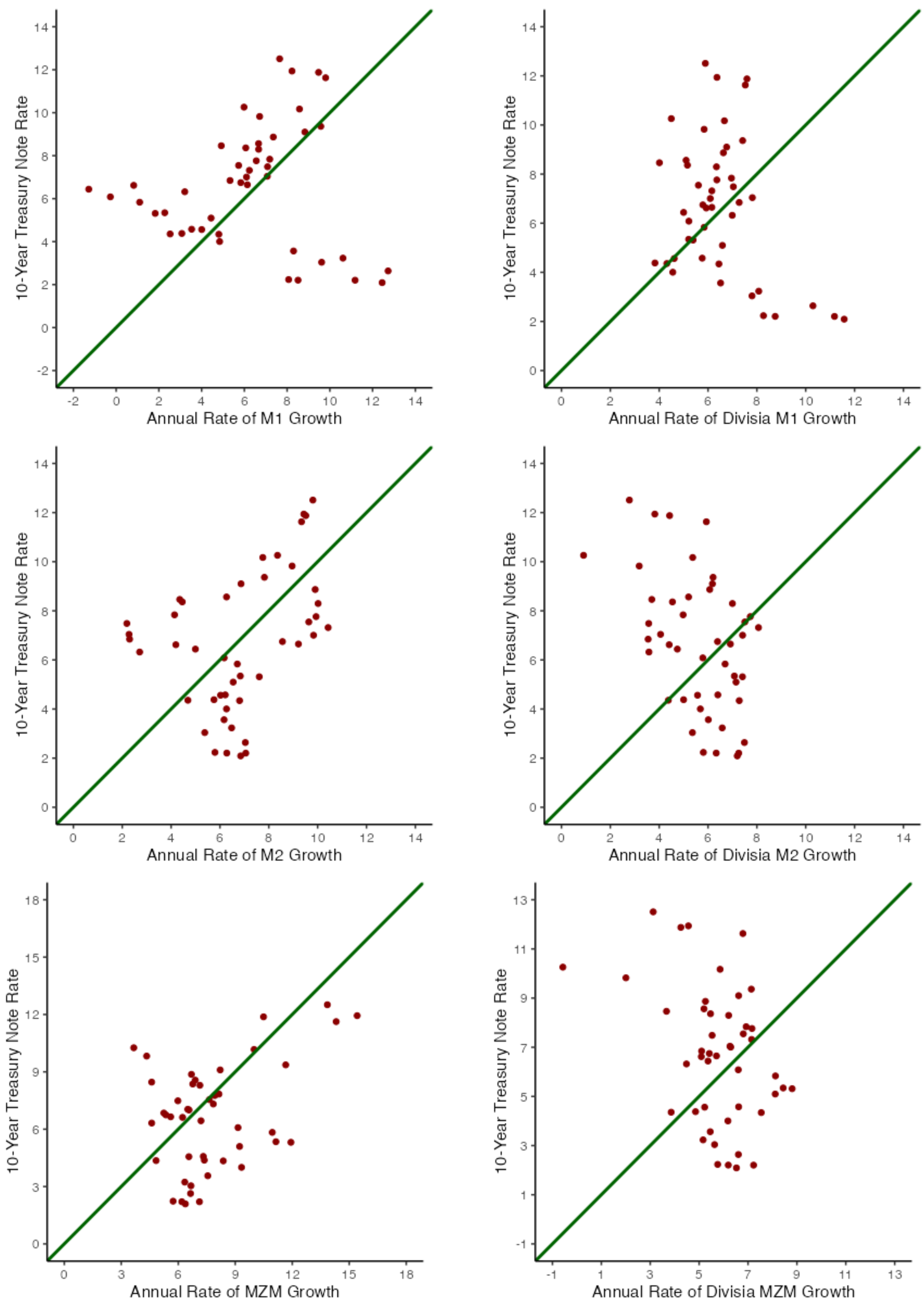
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 8: Scatter Plots of BK Filtered Inflation and the Money Growth (min(6) & max(32)); 1968–2019



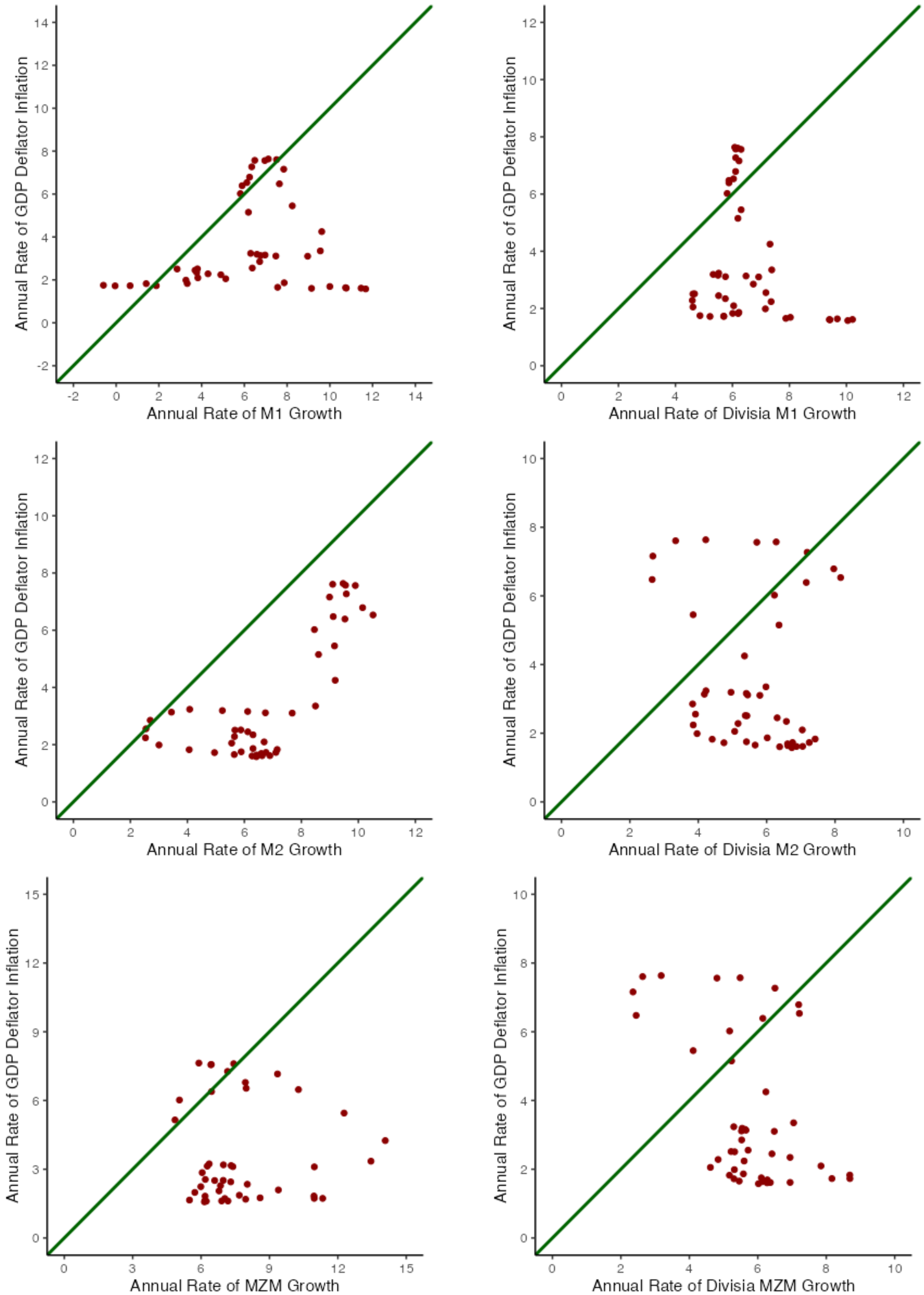
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 9: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(6) & max(32)); 1968–2019



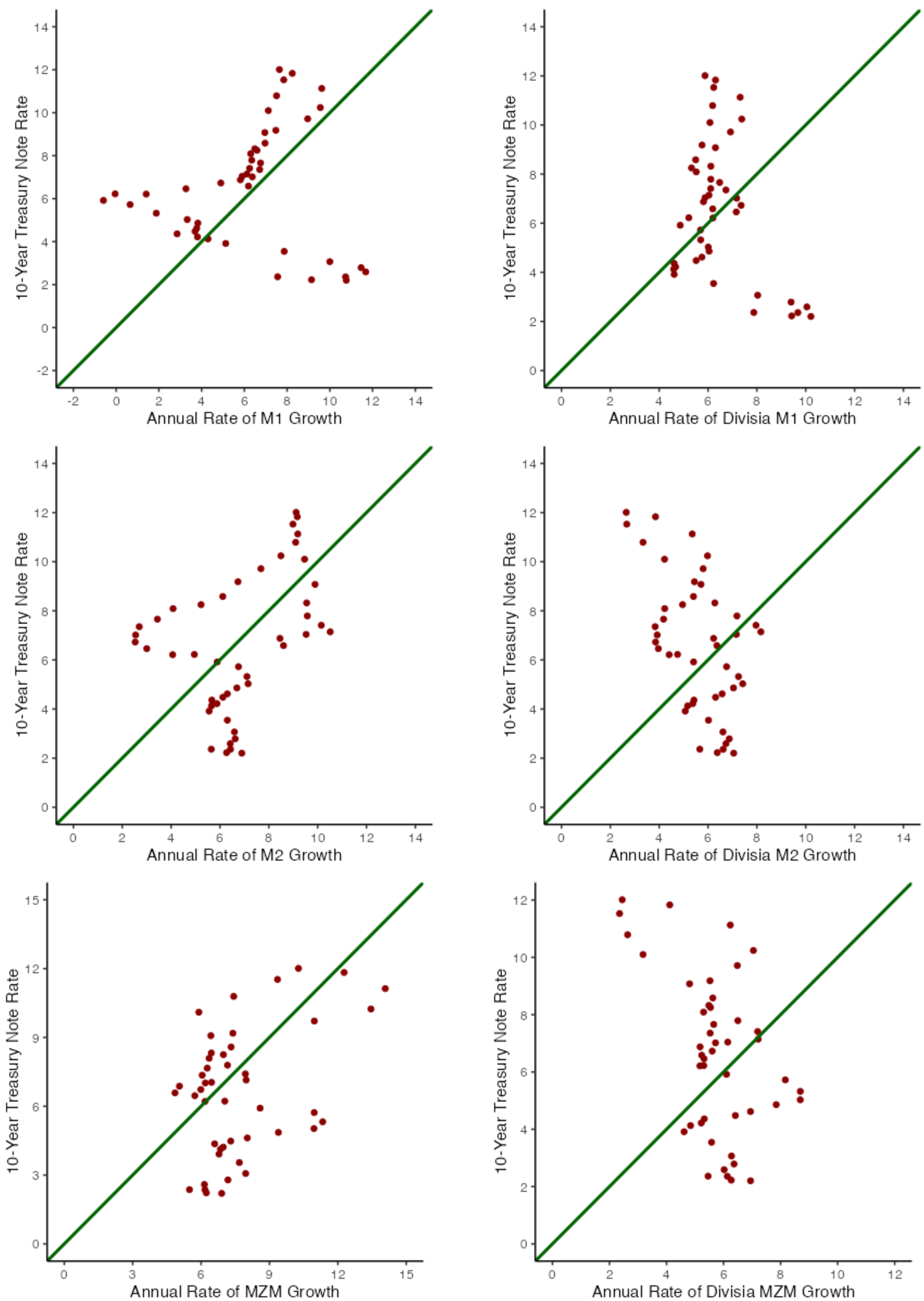
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 10: Scatter Plots of BK Filtered Inflation and the Money Growth (min(2) & max(60)); 1968–2019



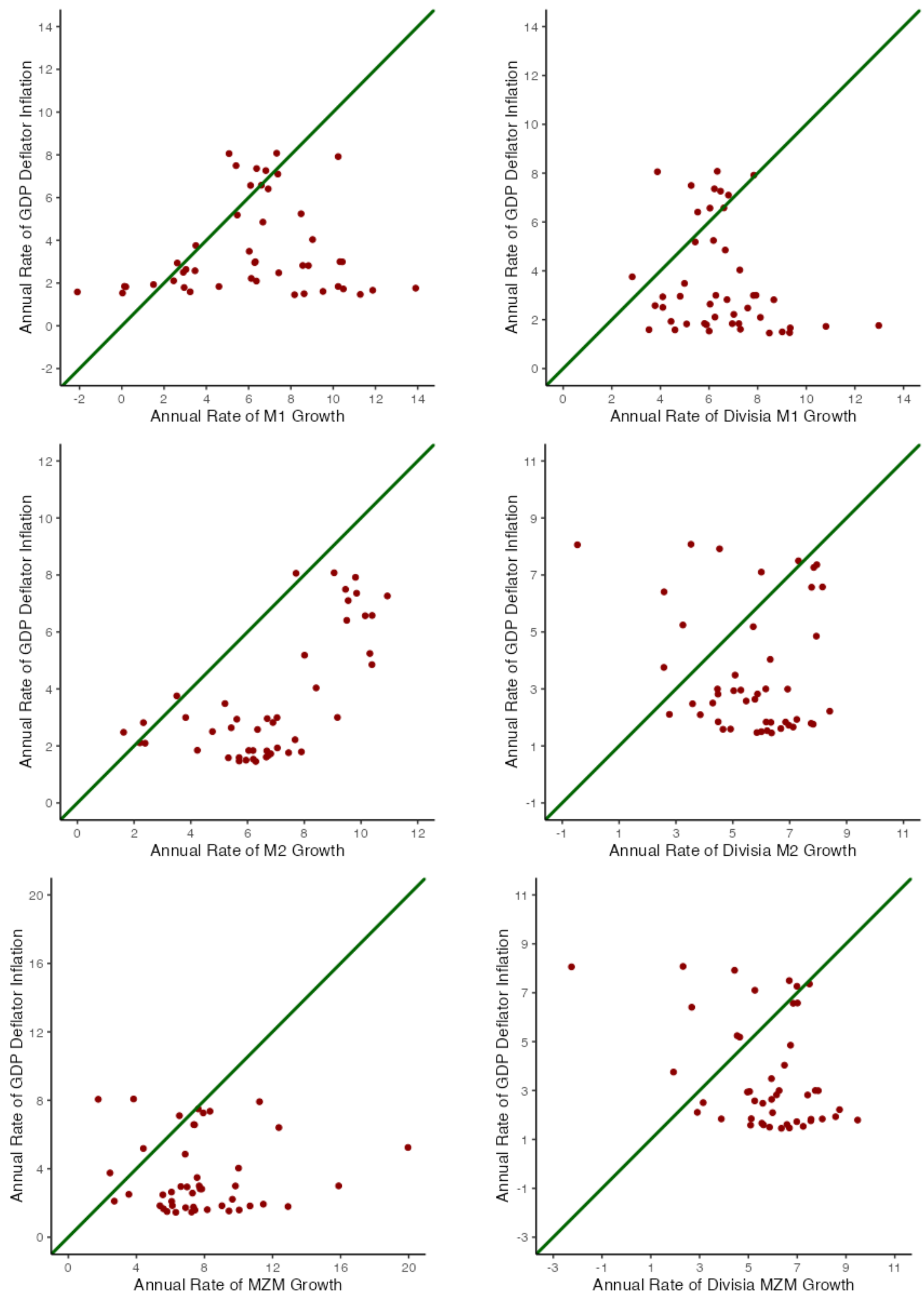
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 11: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(2) & max(60)); 1968–2019



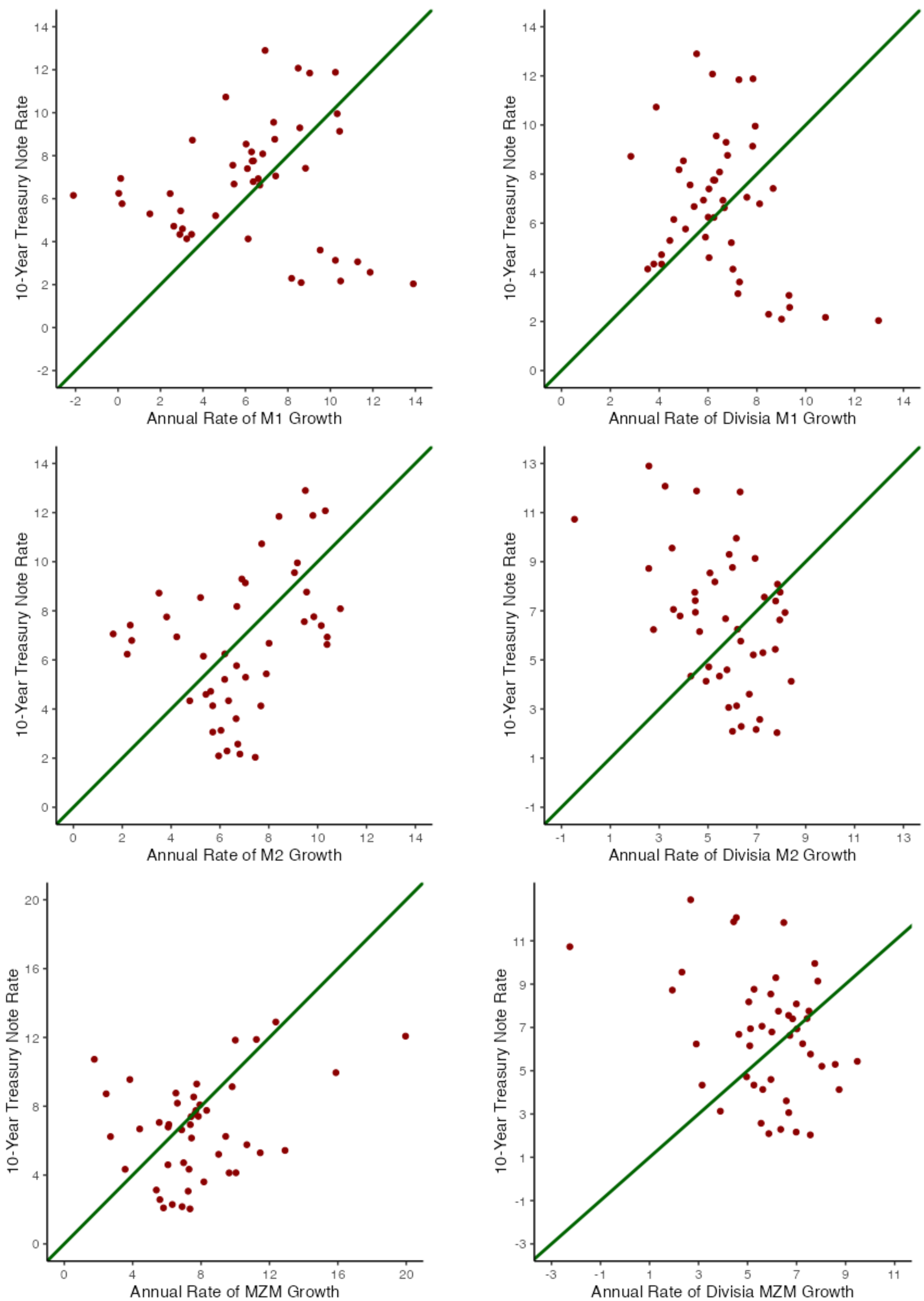
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 12: Scatter Plots of BK Filtered Inflation and the Money Growth (min(8) & max(30)); 1968–2019



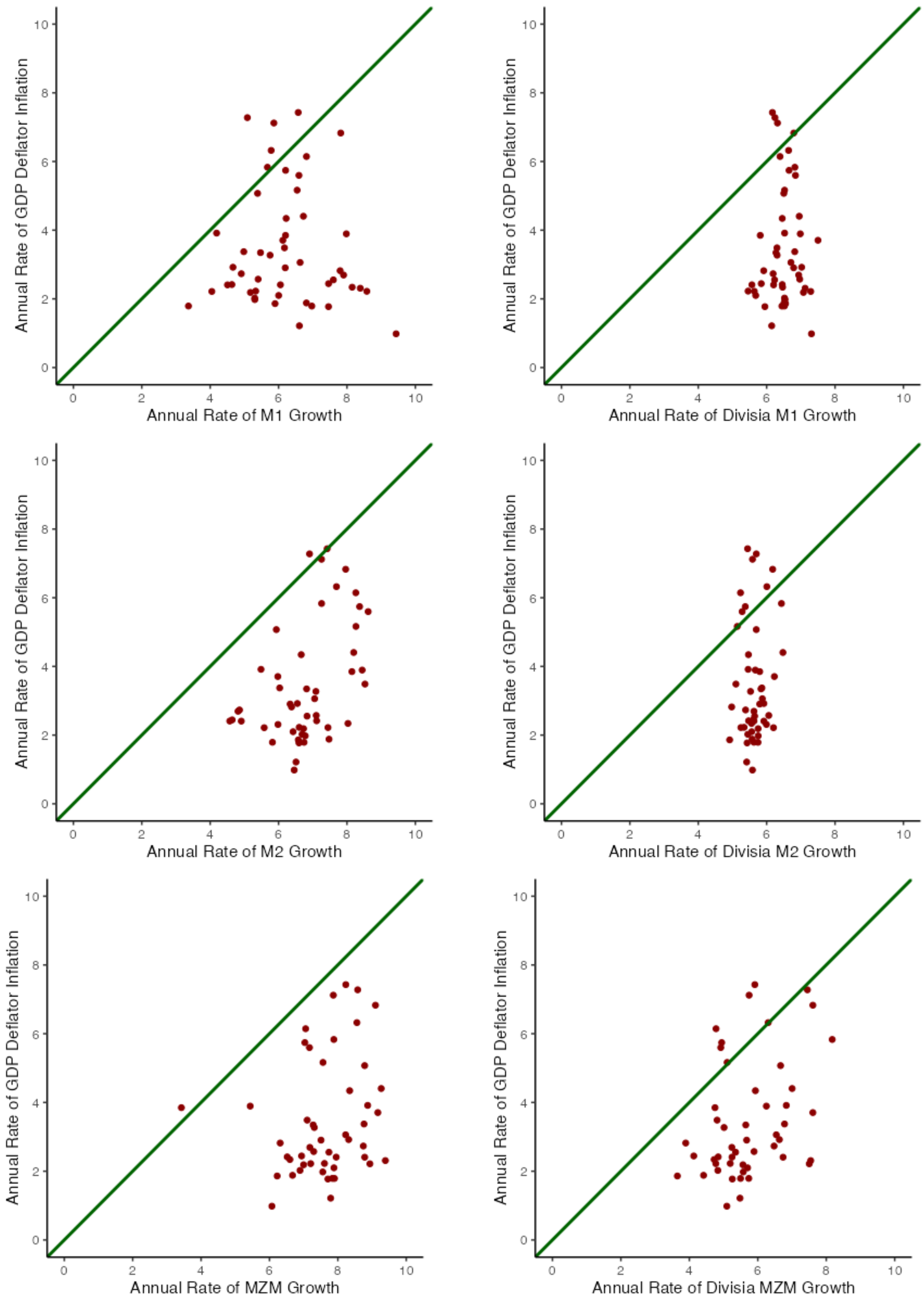
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 13: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(8) & max(30)); 1968–2019



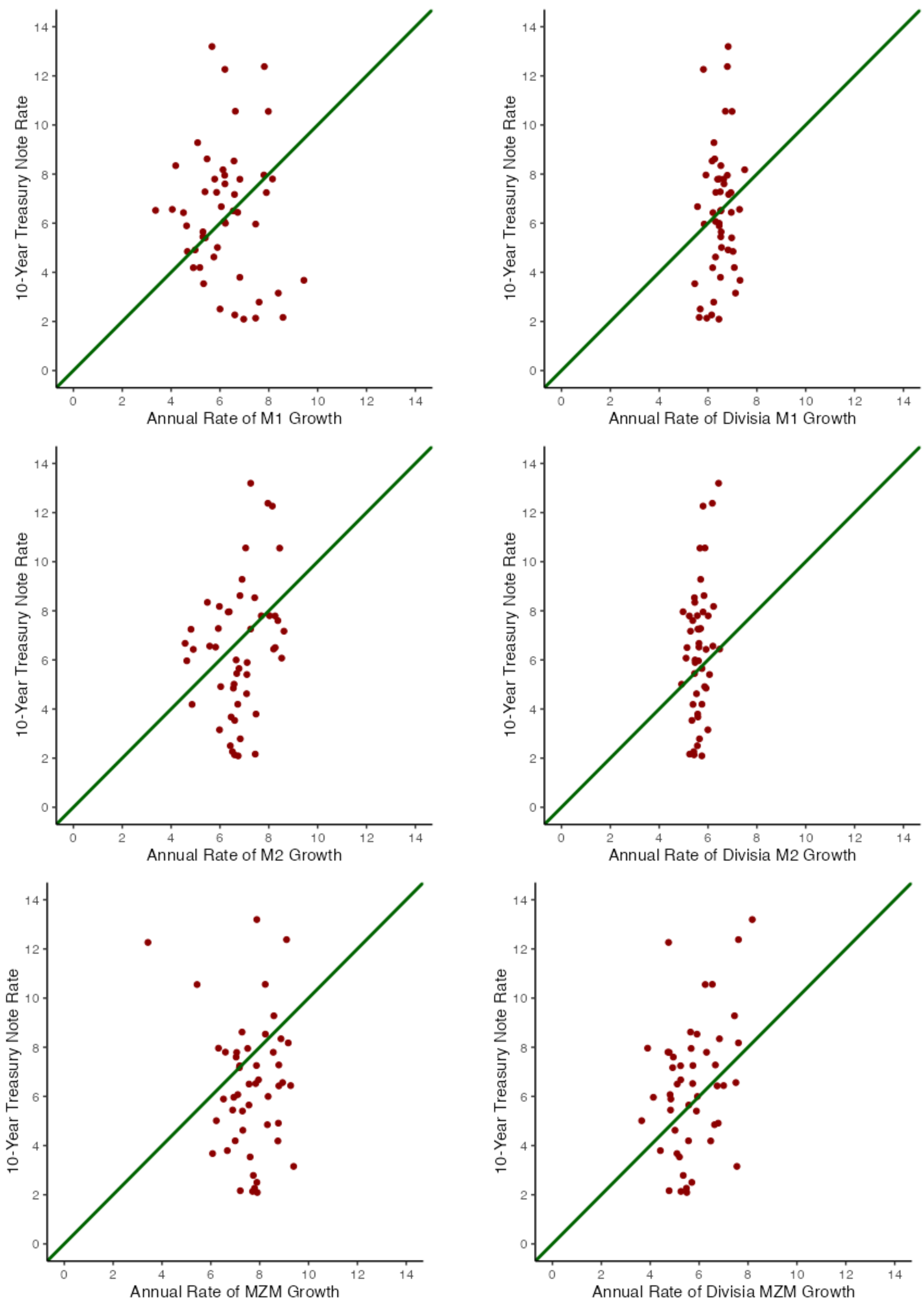
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 14: Scatter Plots of Hamilton Filtered Inflation and the Money Growth; 1968–2019



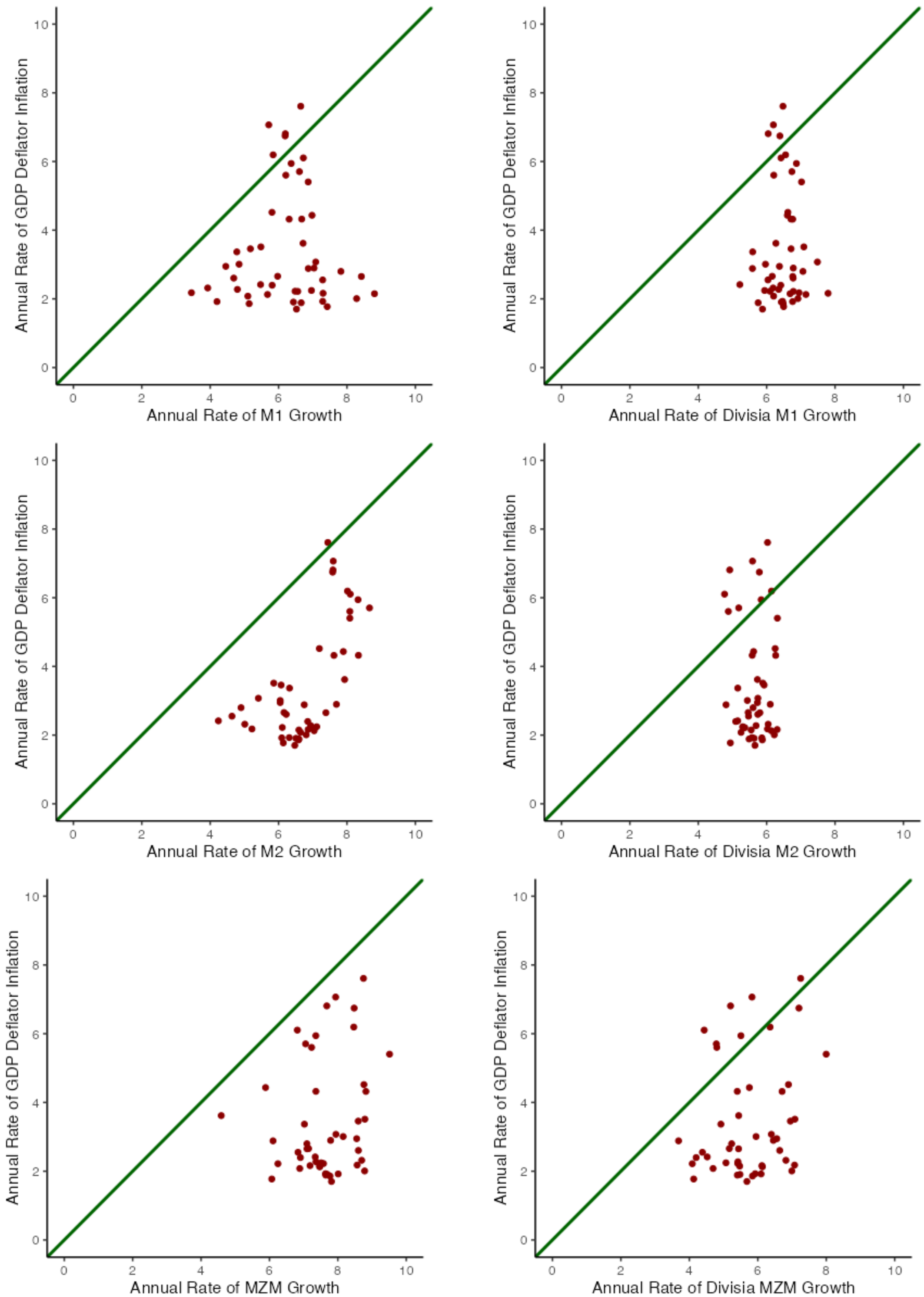
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 15: Scatter Plots of Hamilton Filtered Long-term Interest Rate and the Money Growth; 1968–2019



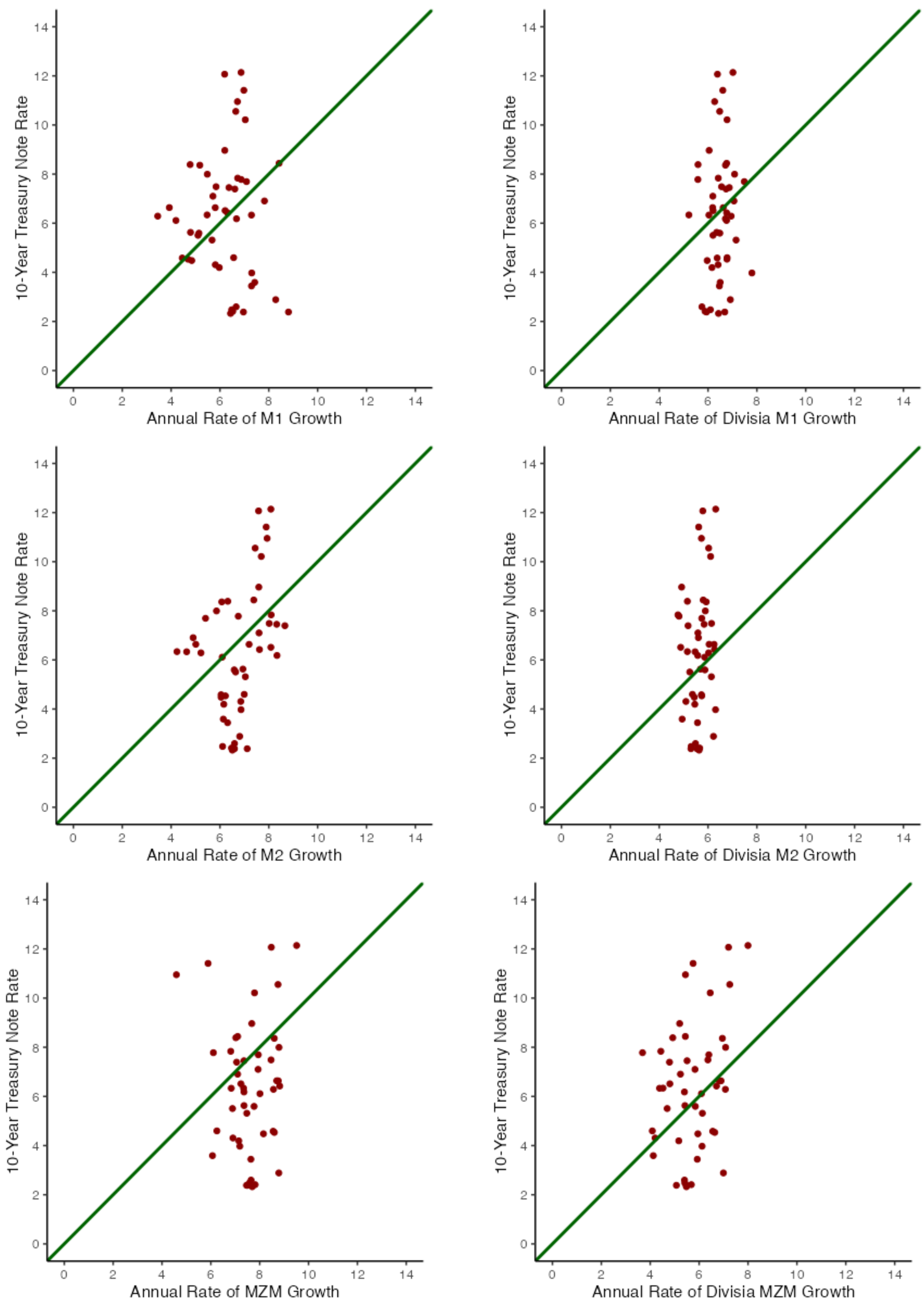
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 16: Scatter Plots of Modified Hamilton Filtered Inflation and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 17: Scatter Plots of Modified Hamilton Filtered Long-term Interest Rate and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

3.2 The Early Subsample: Smoothed Data for 1968–83

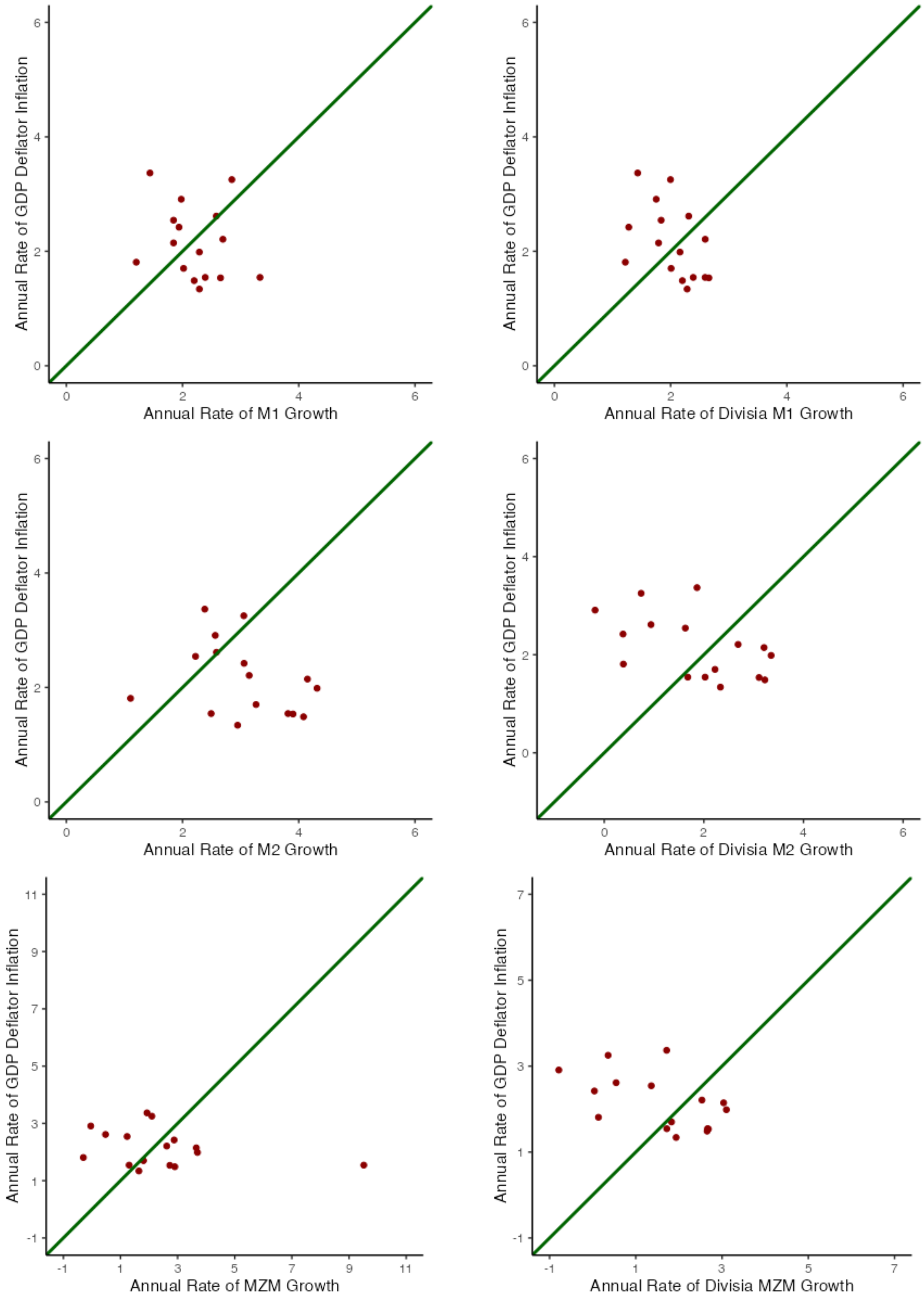
To further analyze the influence of filtering techniques on the results, I conduct a re-examination of the data using two subsamples. The first subsample spans from 1968:1 to 1983:4 and encompasses significant events such as the oil price shock in the early 1970s and the deregulation of interest rates in the early 1980s. This period also witnessed a shift in the Federal Reserve's policy approach towards monetary targeting, characterized by Volcker's tight monetary policies during the late 1970s and early 1980s. The subsample's endpoint includes the recessions of 1980 and 1981-82.

Figures 20 and 21 demonstrate that as the smoothing parameter of the EWMA filter (β) increases, Lucas' proposition holds, as the scatter plots are fitting along the 45° line. Regardless of the measurement used, the M1 and M2 money growth observations are closer to the 45° line, possibly due to the Federal Reserve's emphasis on these measures as targets for monetary policy during the subsample period. The regression coefficients under the EWMA filter in Table 2 support this observation, indicating a positive relationship between money growth and inflation, and money growth and interest rates, especially for simple-sum M1 and Divisia M1.

Figures 22 to 27 present the scatter plots generated using the BK filter under different window ranges. The observations deviate from the 45° line, similar to the EWMA filter. However, the observations gradually approach the 45° line as the window range widens. The regression coefficients in the BK filter column of Table 2 reinforce this pattern, showing limited positive relationships between money growth and inflation for most money aggregates. Nevertheless, when considering simple-sum M1, MZM, and Divisia M1 aggregates, a positive relationship emerges between interest rates and money growth, as well as between inflation rates and money growth.

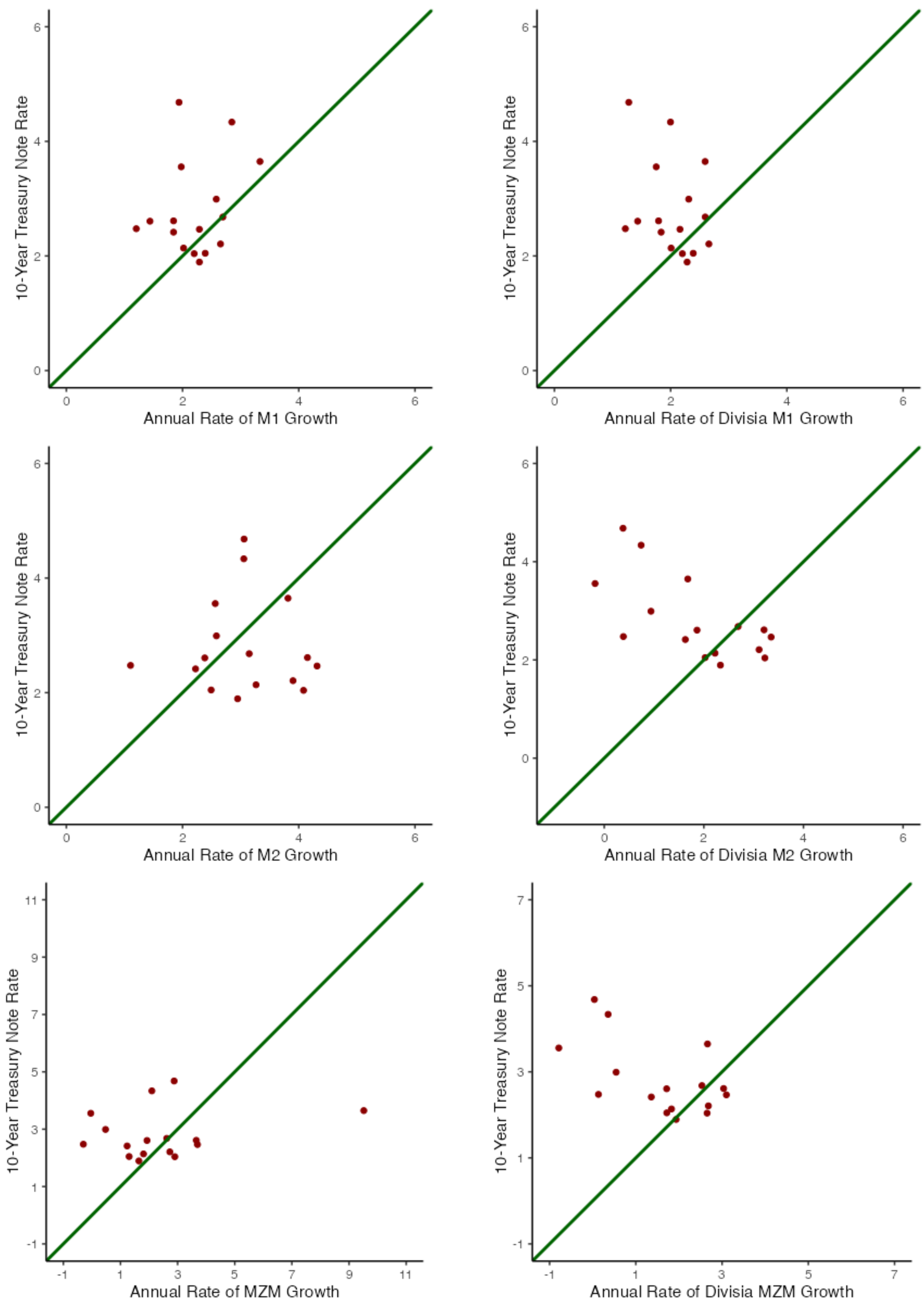
In Figures 28 and 29, the scatter plots produced by the Hamilton filter reveal that Lucas' proposition holds when using the Divisia M2 and MZM series, but not for Divisia M1 and simple-sum series. The regression coefficients in the Hamilton filter column of Table 2 support these findings, indicating a strong positive relationship between interest rates and money growth, as well as between inflation rates and money growth, for the Divisia M2 and MZM coefficients. Conversely, the modified Hamilton filter in Figures 30 and 31 shows that only the scatter plots for Divisia and simple-sum MZM somewhat align with the quantity-theoretic prediction, albeit not as prominently as the plots from the EWMA filter. Once again, the regression coefficients in the modified Hamilton filter column of Table 2 display a strong positive relationship between money growth and interest rates, particularly for simple-sum and Divisia MZM.

Figure 18: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.5$); 1968–83



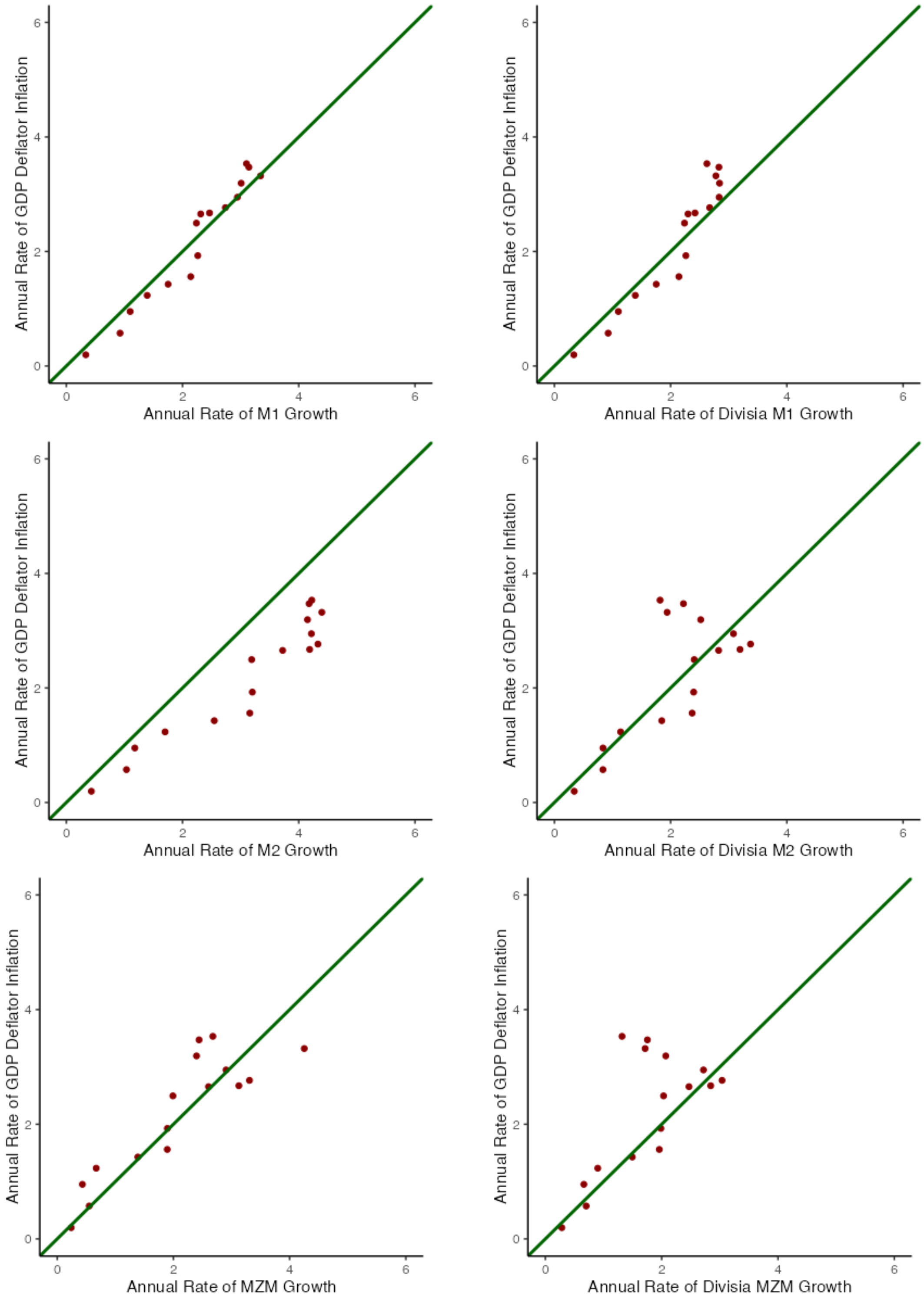
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 19: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.5$); 1968–83



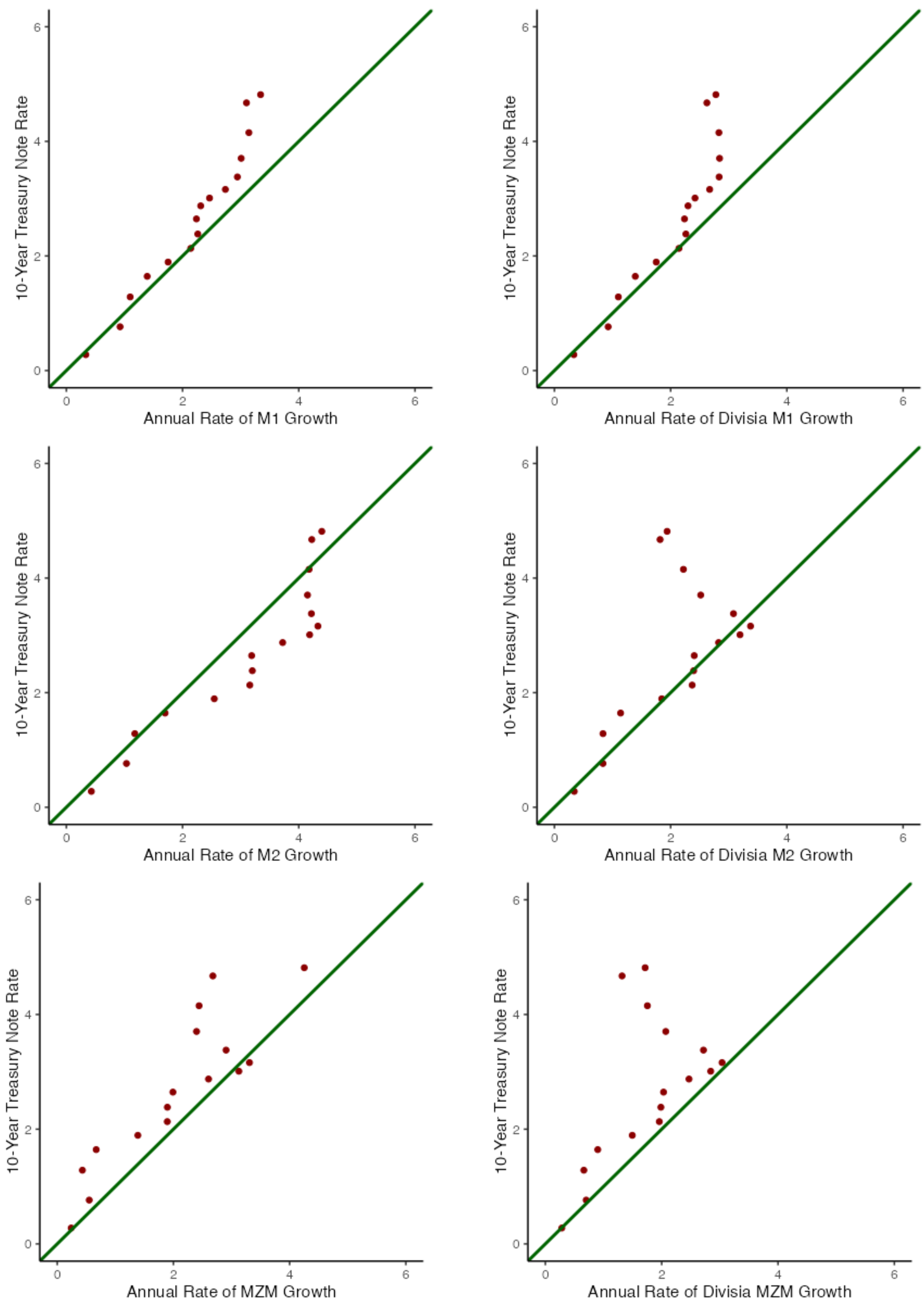
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 20: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.95$); 1968–83



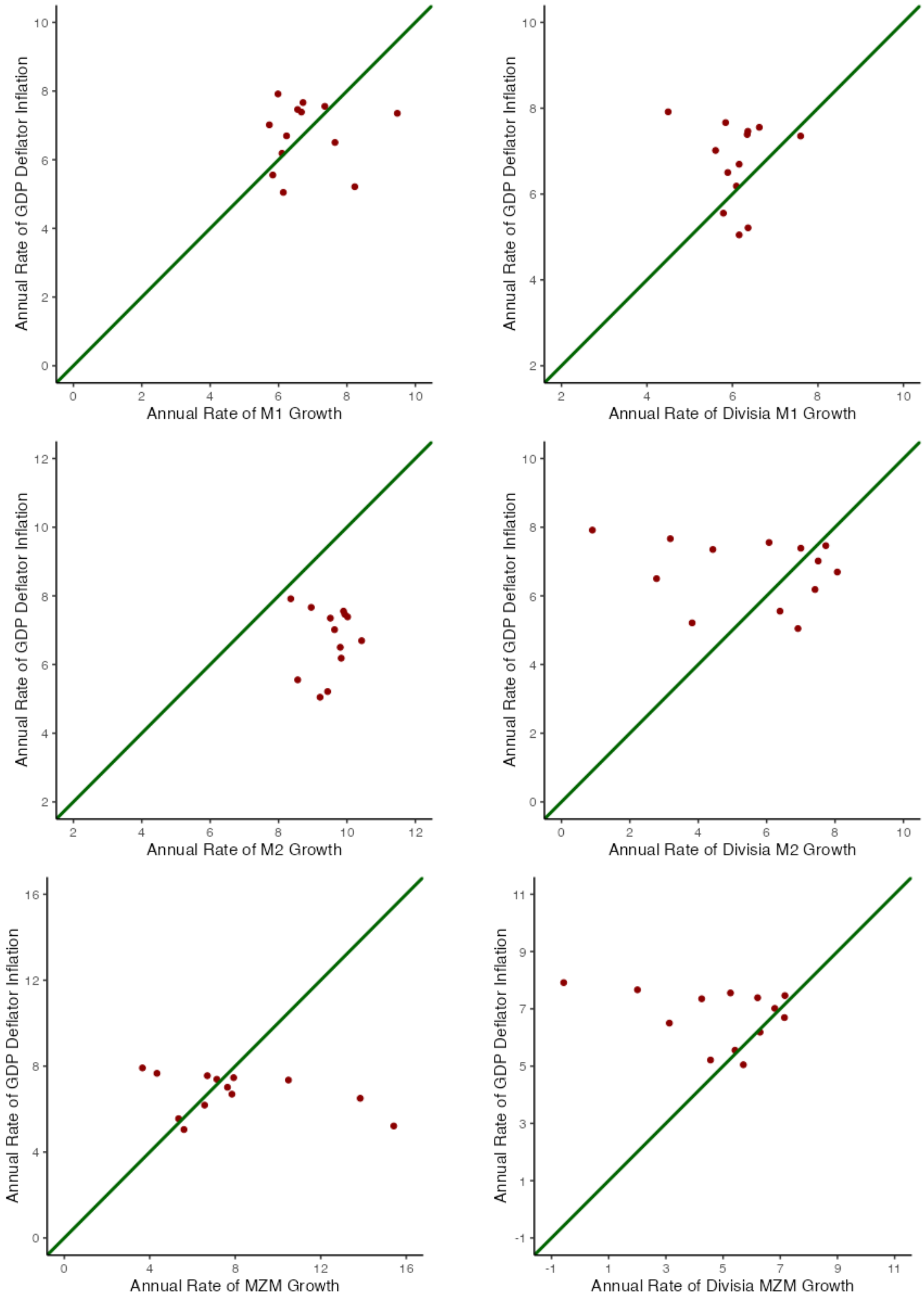
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 21: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.95$); 1968–83



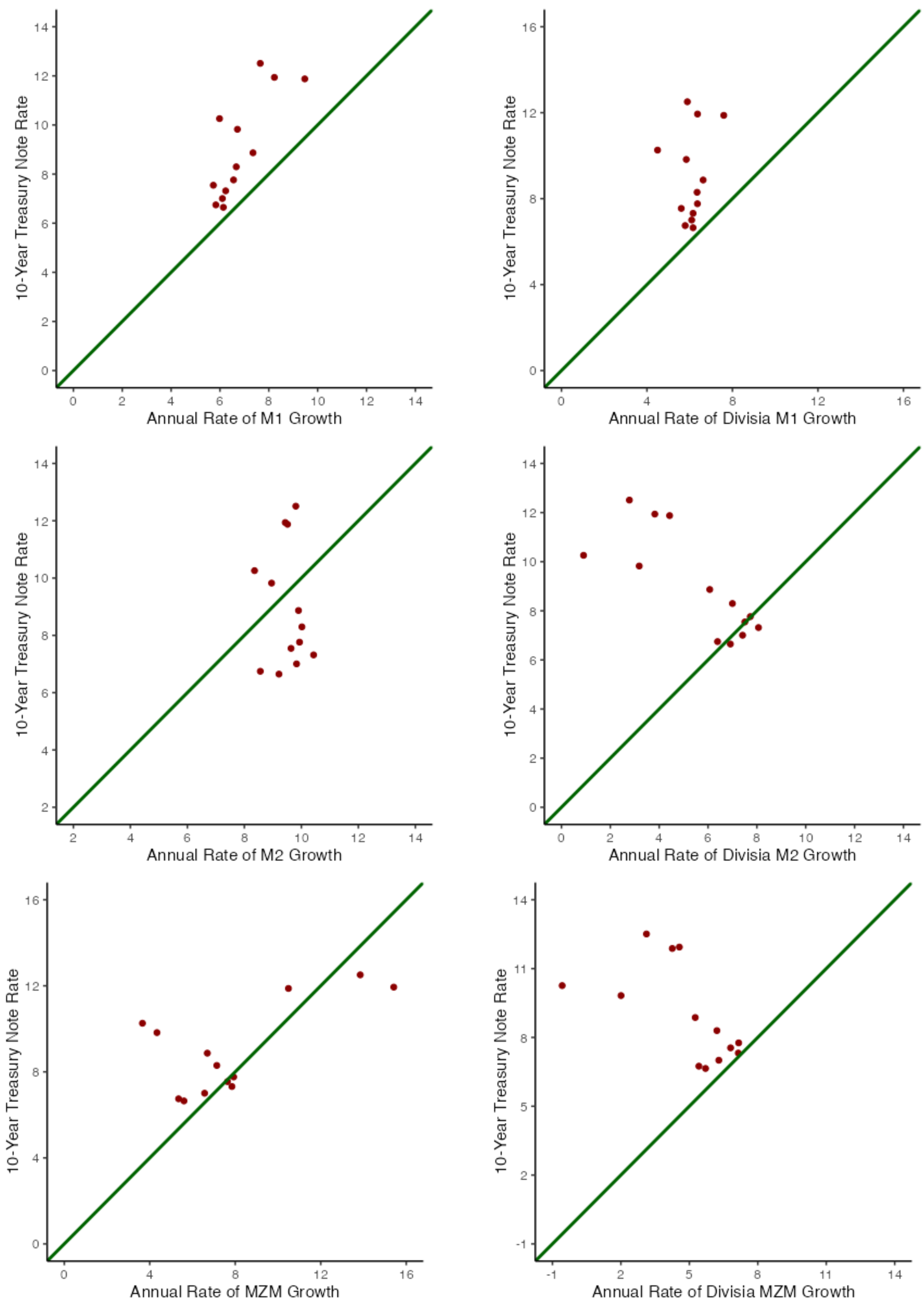
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 22: Scatter Plots of BK Filtered Inflation and the Money Growth (min(6) & max(32)); 1968–83



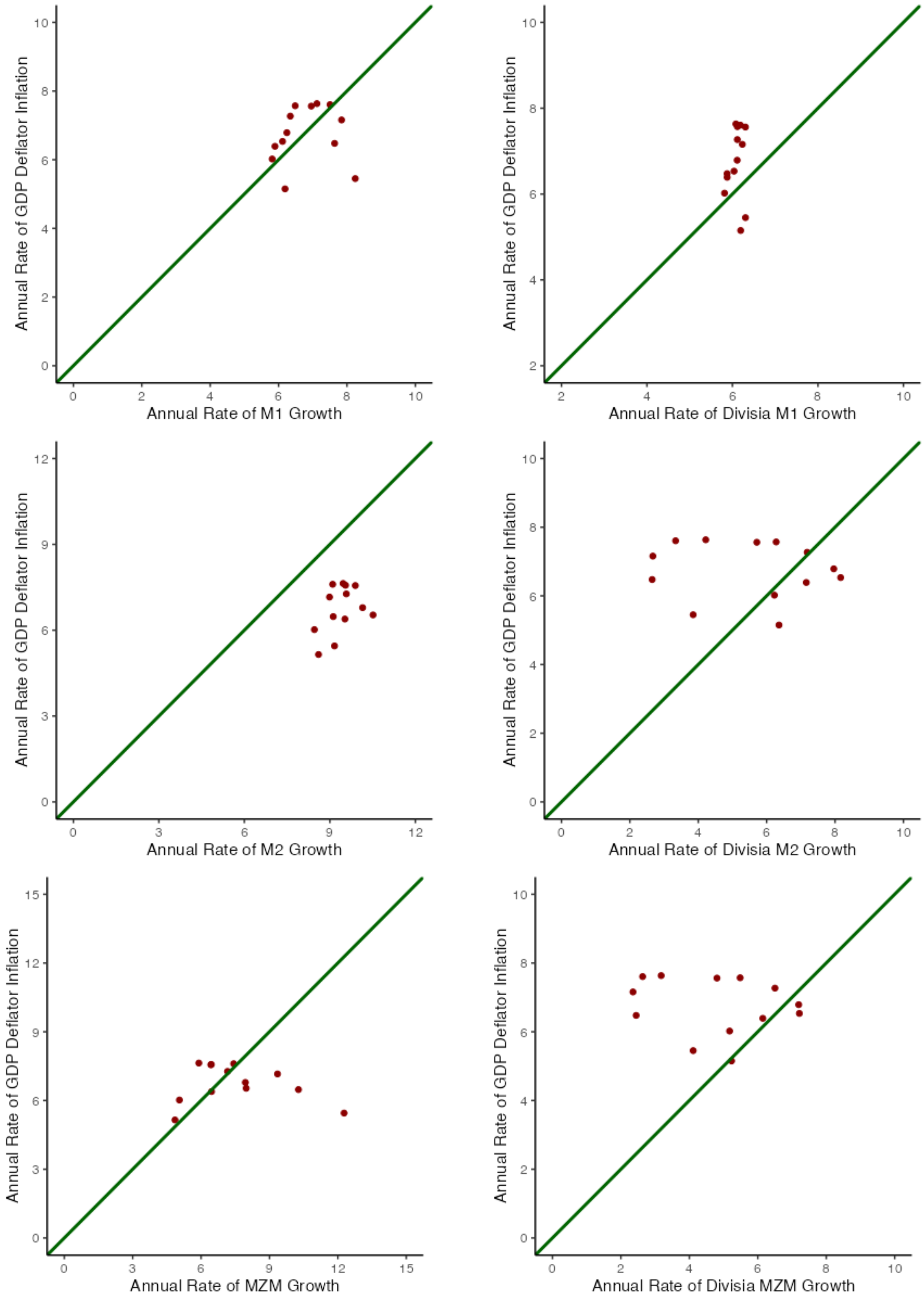
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 23: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(6) & max(32)); 1968–83



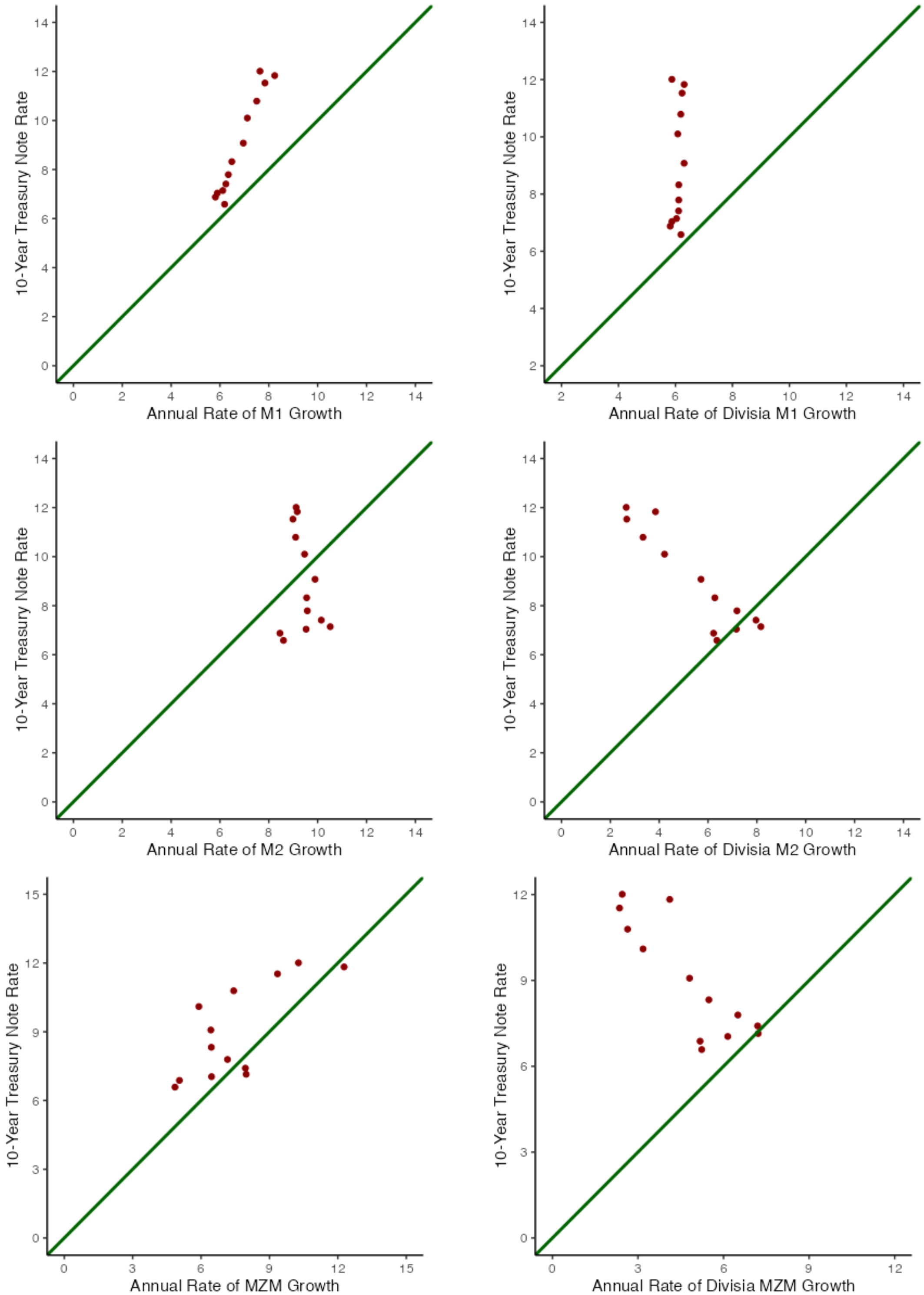
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 24: Scatter Plots of BK Filtered Inflation and the Money Growth (min(2) & max(60)); 1968–83



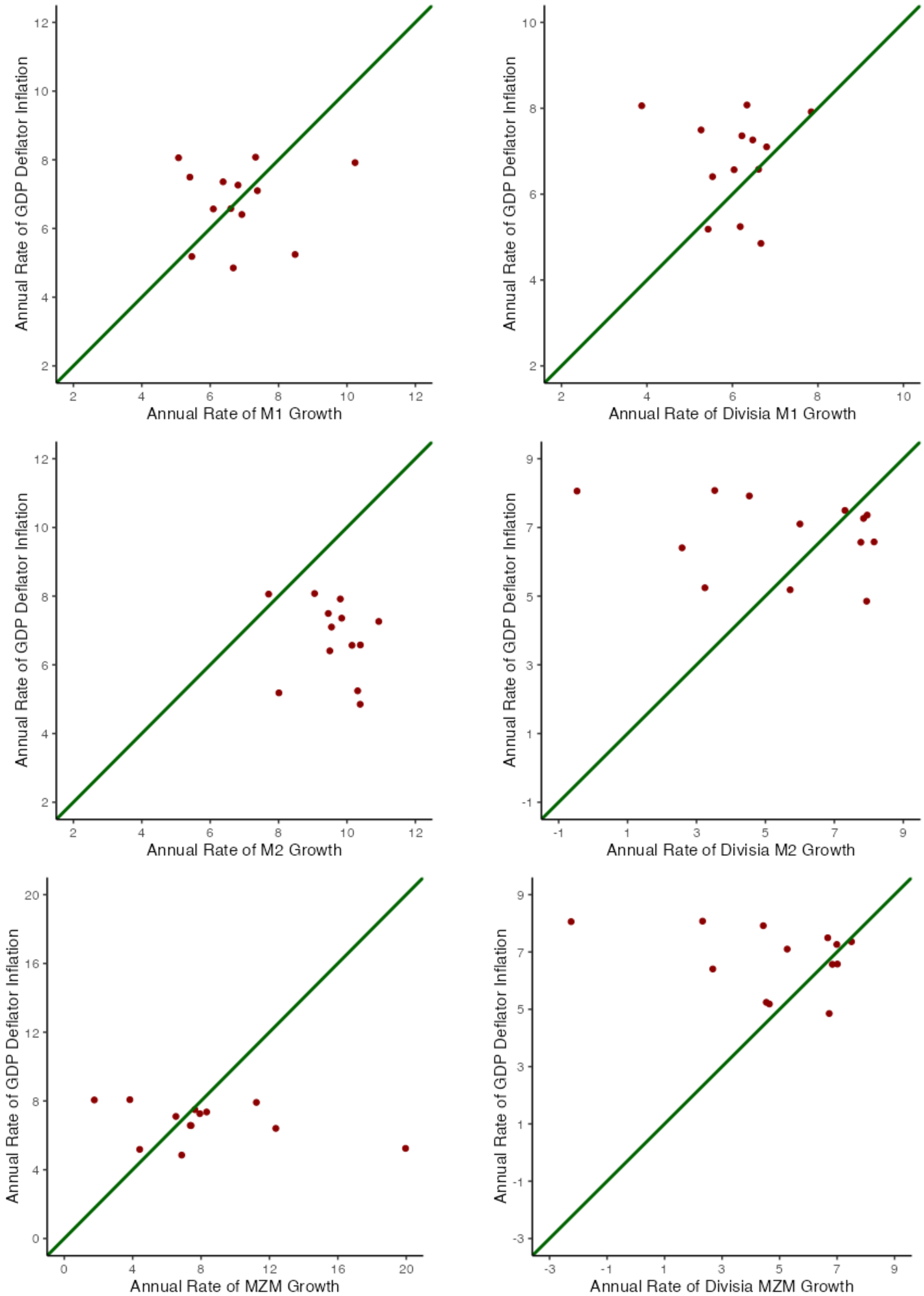
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 25: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(2) & max(60)); 1968–83



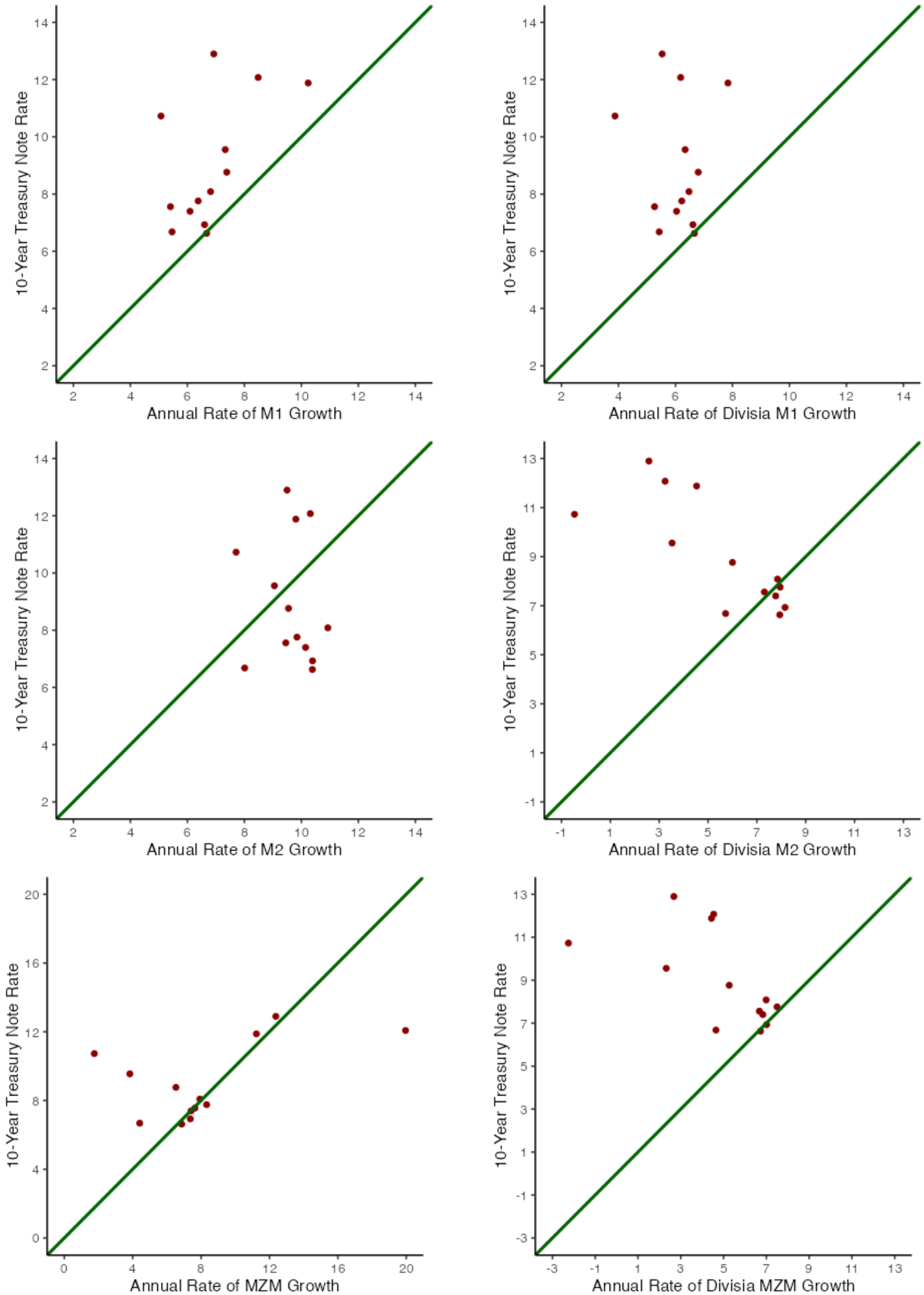
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 26: Scatter Plots of BK Filtered Inflation and the Money Growth (min(8) & max(30)); 1968–83



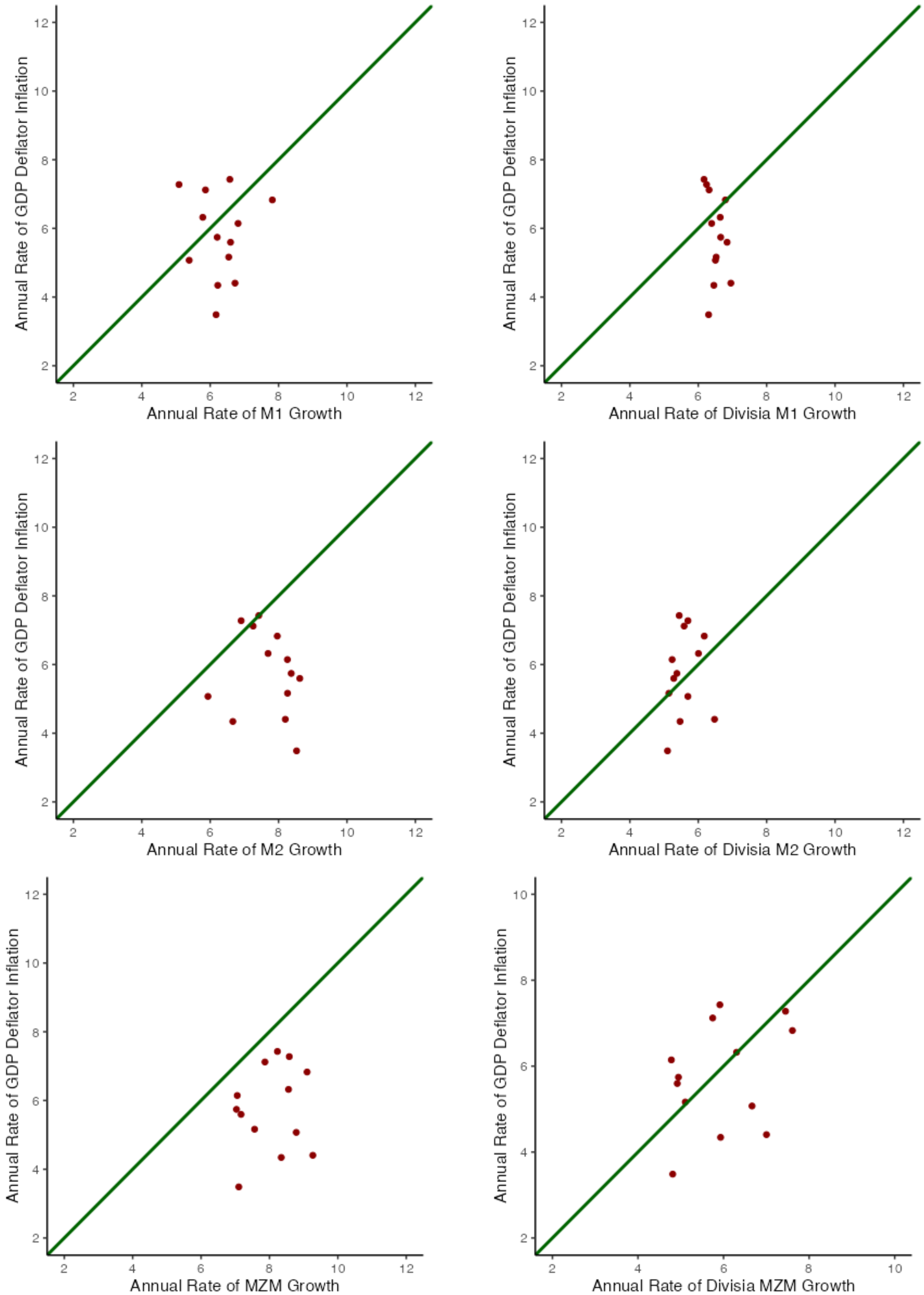
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 27: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(8) & max(30)); 1968–83



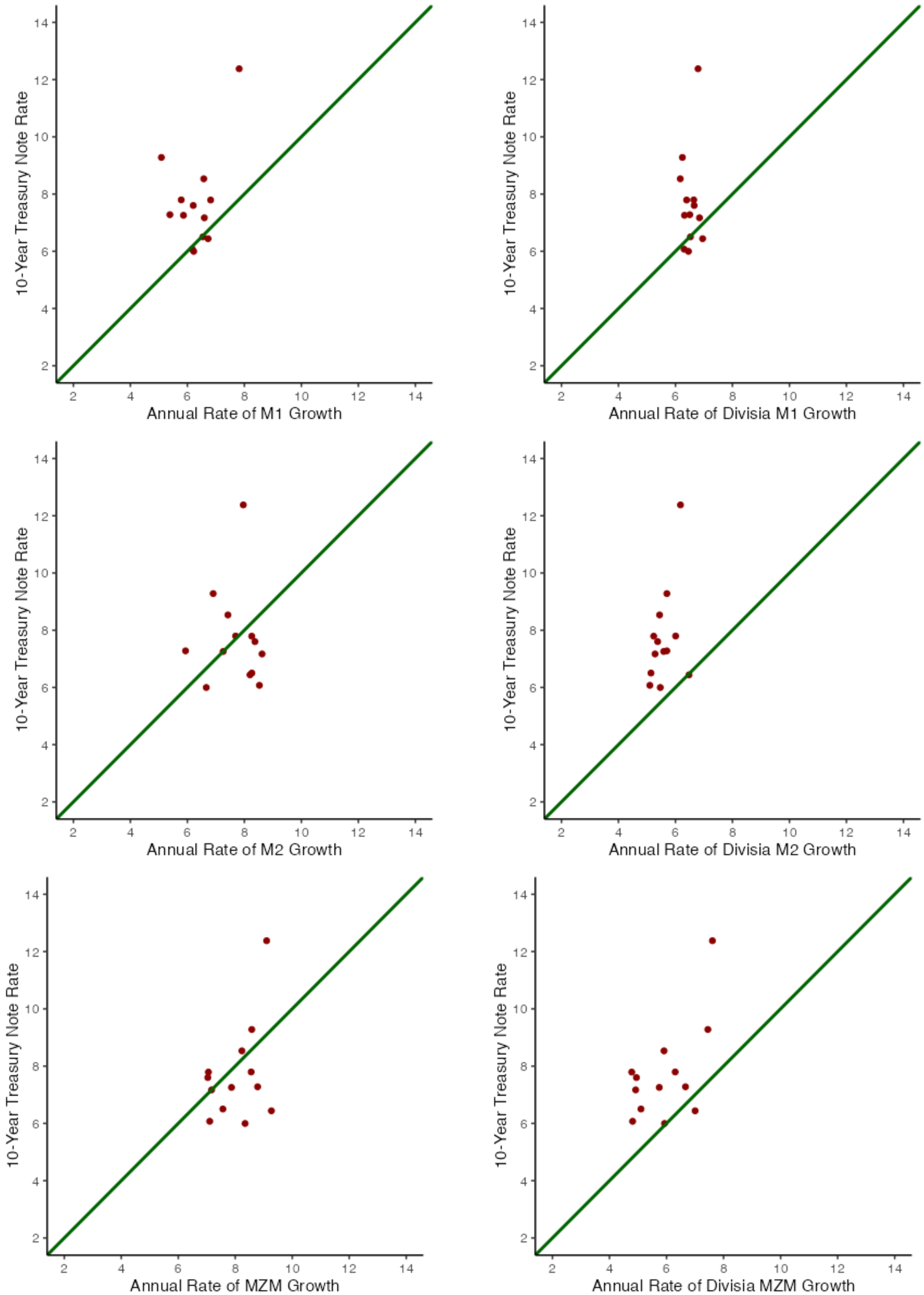
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 28: Scatter Plots of Hamilton Filtered Inflation and the Money Growth; 1968–83



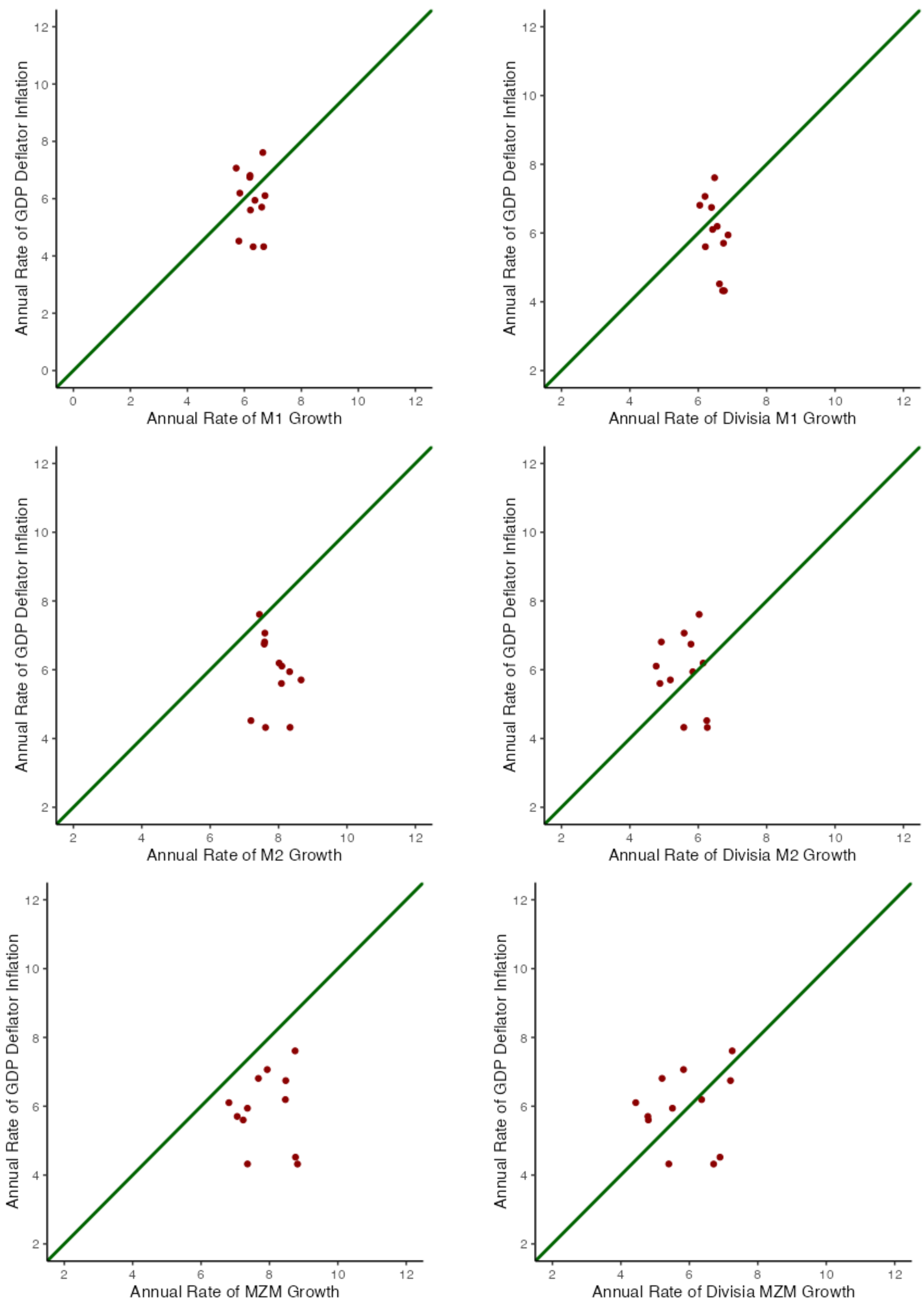
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 29: Scatter Plots of Hamilton Filtered Long-term Interest Rate and the Money Growth; 1968–83



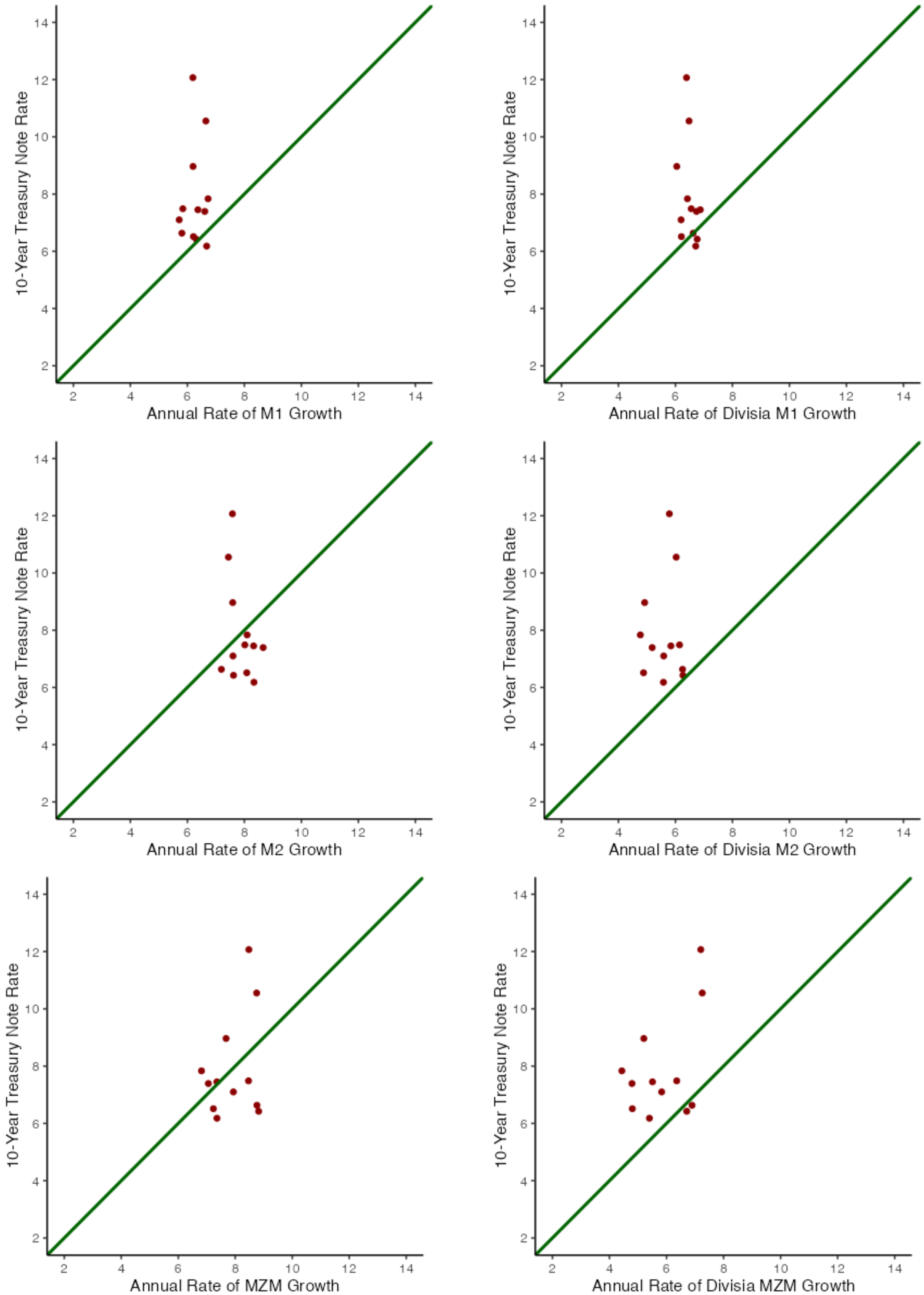
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 30: Scatter Plots of Modified Hamilton Filtered Inflation and the Money Growth; 1968–83



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 31: Scatter Plots of Modified Hamilton Filtered Long-term Interest Rate and the Money Growth; 1968–83



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Table 2: Coefficients of the Regressions on Filtered Data, 1968–1983

Variable	EWMA Filter		BK Filter		Hamilton Filter		Modified Hamilton Filter	
	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate
M1	1.17***	1.45***	0.02	0.94***	–0.13	0.33	–0.22	0.73
	(0.04)	(0.05)	(0.13)	(0.13)	(0.37)	(0.50)	(0.44)	(0.77)
M2	0.76***	0.91***	0.09	–0.55*	–0.36*	–0.33	–0.25	–1.15**
	(0.03)	(0.05)	(0.20)	(0.30)	(0.20)	(0.28)	(0.32)	(0.53)
MZM	0.76***	0.96***	–0.10*	0.33***	0.34	0.71**	0.06	0.80**
	(0.06)	(0.07)	(0.04)	(0.06)	(0.22)	(0.29)	(0.21)	(0.35)
Divisia M1	1.32***	1.58***	–0.04	0.55*	–0.91	–0.33	–2.55***	–2.41***
	(0.06)	(0.09)	(0.19)	(0.30)	(0.82)	(1.12)	(0.49)	(1.02)
Divisia M2	0.84***	0.87***	–0.07	–0.77***	0.98**	1.50**	–0.47	0.17
	(0.1)	(0.15)	(0.06)	(0.08)	(0.46)	(0.62)	(0.28)	(0.51)
Divisia MZM	0.87***	0.89***	–0.11*	–0.50***	0.55***	1.11***	0.10	0.93***
	(0.12)	(0.17)	(0.06)	(0.10)	(0.17)	(0.20)	(0.16)	(0.25)

Notes: *** p<0.01, **p<0.05, *p<0.10. Standard errors in parenthesis. Inflation counts for GDP deflator; Interest rate counts for 10–Year Treasury Note

3.3 The Recent Subsample: Smoothed Data for 1984–2019

The second subsample focuses on a period marked by significant monetary policies and economic events that shaped money supply, inflation, and interest rates in the U.S. Under the leadership of Alan Greenspan, the Federal Reserve adopted a stance of providing necessary monetary accommodation. This period witnessed implicit inflation targeting in the 1990s and the implementation of quantitative easing following the global financial crisis of 2008. Forward guidance also became a prominent tool, influencing market expectations of future interest rates. Economic events such as the dot-com bubble burst in the late 1990s, the global financial crisis in 2008, and the European sovereign debt crisis profoundly impacted the economy, money supply growth, and interest rates.

Figures 32 to 35 display scatter plots generated using the EWMA filter. As the smoothing parameter (β) increases, the scatter plots progressively align closer to the 45° line. Notably, the scatter plots for both money growth and inflation, as well as money growth and the interest rate, in Figures 34 and 35 respectively, appear similar for both the simple-sum and Divisia aggregates. However, the regression coefficients under the EWMA filter in Table 3 reveal that a positive relationship between money growth and inflation, as well as money growth and interest rates, is only evident for the simple-sum aggregates.

Using the BK filter, Figures 36 to 41 show that the scatter plots become closer to the 45° line for wider window ranges, and they exhibit a similar pattern for both simple-sum and Divisia aggregates. The regression coefficients further show that in Table 3, irrespective of money growth measurement, there are no positive relationships between money growth and inflation and money growth and interest rate. However, simple-sum MZM shows a positive relationship between interest rate and money growth and inflation rate and money growth.

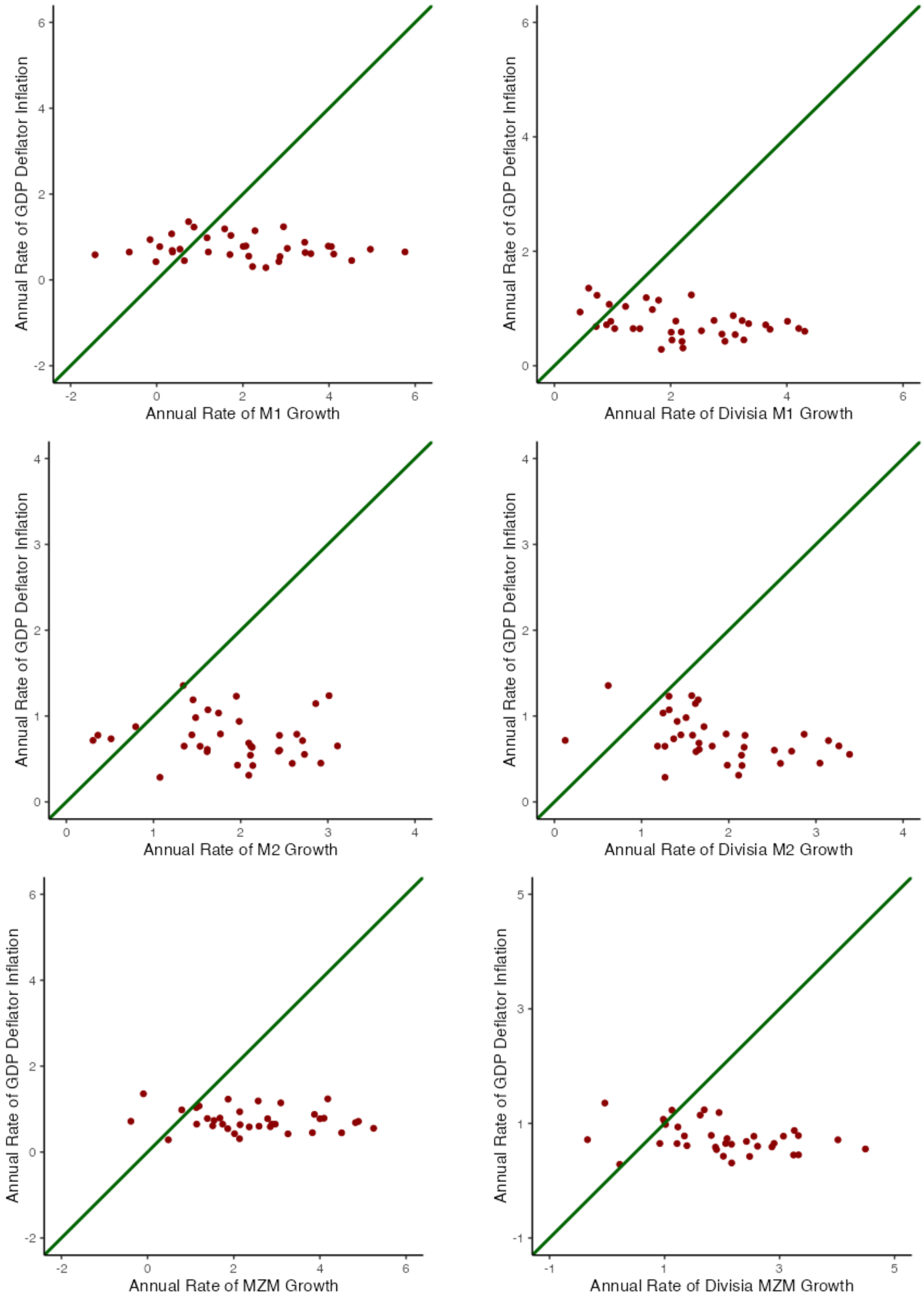
Figures 42 and 43, employing the Hamilton filter, illustrate scatter plots where the observations align closer to the 45° line for money growth and interest rates, and they are similar for both simple-sum and Divisia aggregates. The corresponding regression coefficients in Table 3 demonstrate a strong positive relationship between money growth and the interest rates and money growth and inflation rates across all Divisia aggregate money supply. In contrast, such a strong relationship is only present for the simple-sum M2 aggregate. Similarly, the modified Hamilton filter in Figures 44 and 45 exhibits scatter plots closer to the 45° line for money growth and interest rates, displaying a similar pattern for both simple-sum and Divisia aggregates. However, only regression coefficients derived from the Divisia aggregates and simple-sum M2 produce quantity-theoretic predictions.

Table 3: Coefficients of the Regressions on Filtered Data, 1984–2019

Variable	EWMA Filter		BK Filter		Hamilton Filter		Modified Hamilton Filter	
	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate	Inflation	Interest Rate
M1	0.14***	0.05	0.02	–0.07	–0.05	–0.19	0.01	–0.12
	(0.04)	(0.09)	(0.02)	(0.06)	(0.05)	(0.15)	(0.05)	(0.17)
M2	0.53***	0.65***	–0.001	–0.06	0.14**	0.53**	0.25***	0.75**
	(0.06)	(0.14)	(0.04)	(0.13)	(0.07)	(0.23)	(0.07)	(0.24)
MZM	0.47***	0.95***	0.06**	0.40***	–0.02	–0.57***	0.02	–0.40*
	(0.06)	(0.12)	(0.02)	(0.08)	(0.06)	(0.18)	(0.07)	(0.23)
Divisia M1	–0.25***	–0.83***	–0.12***	–0.45***	0.27**	0.91**	0.18	0.82**
	(0.09)	(0.17)	(0.03)	(0.11)	(0.13)	(0.42)	(0.12)	(0.40)
Divisia M2	–0.85***	–2.09***	–0.22***	–0.84**	0.92***	2.66***	0.41**	1.96***
	(0.09)	(0.17)	(0.05)	(0.16)	(0.20)	(0.66)	(0.17)	(0.60)
Divisia MZM	–0.74***	–1.19***	–0.11***	–0.06	0.31***	0.59***	0.27***	0.68***
	(0.09)	(0.21)	(0.05)	(0.17)	(0.06)	(0.21)	(0.06)	(0.22)

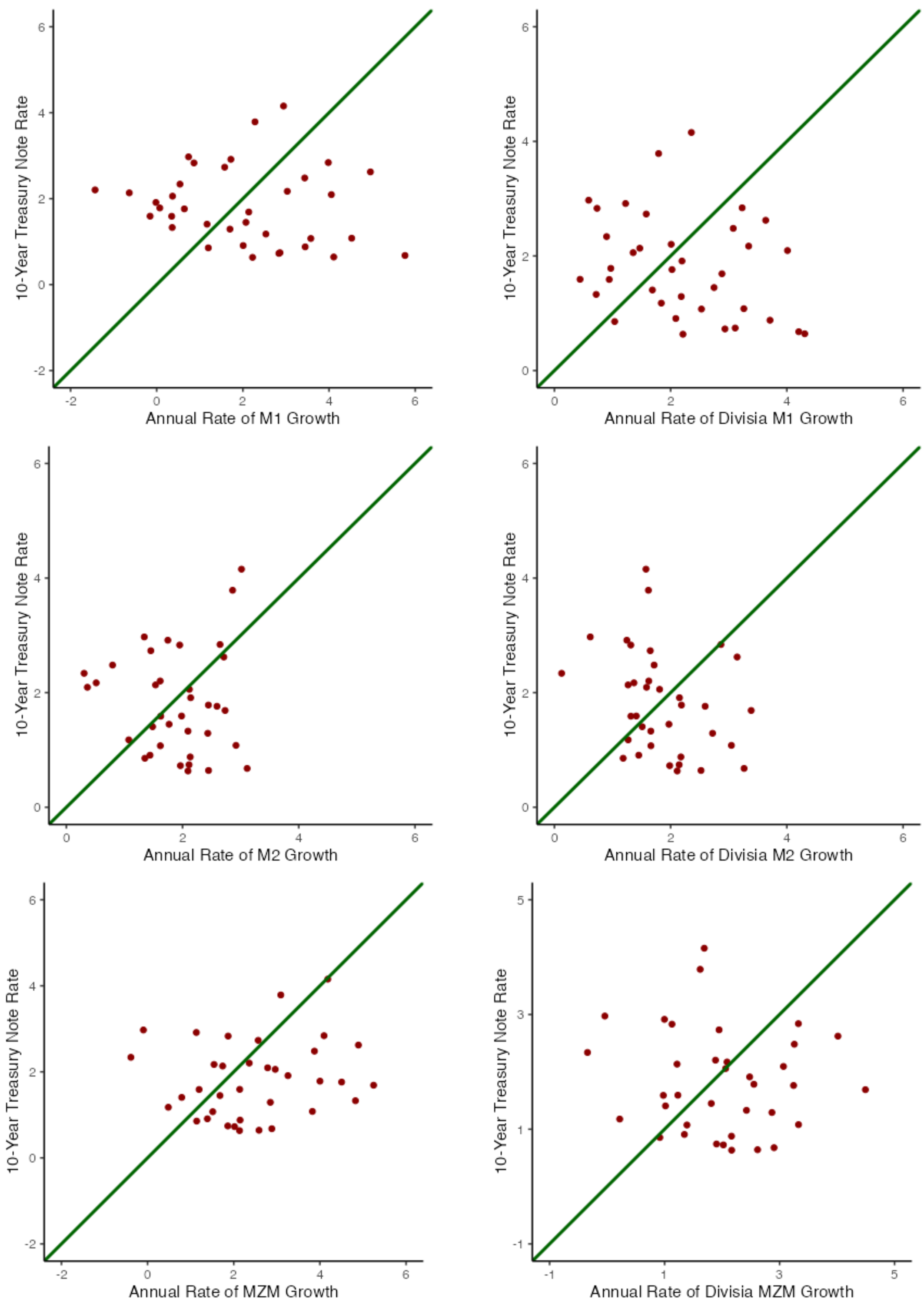
Notes: *** p<0.01, **p<0.05, *p<0.10. Standard errors in parenthesis. Inflation counts for GDP deflator; Interest rate counts for 10–Year Treasury Note

Figure 32: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.5$); 1984–2019



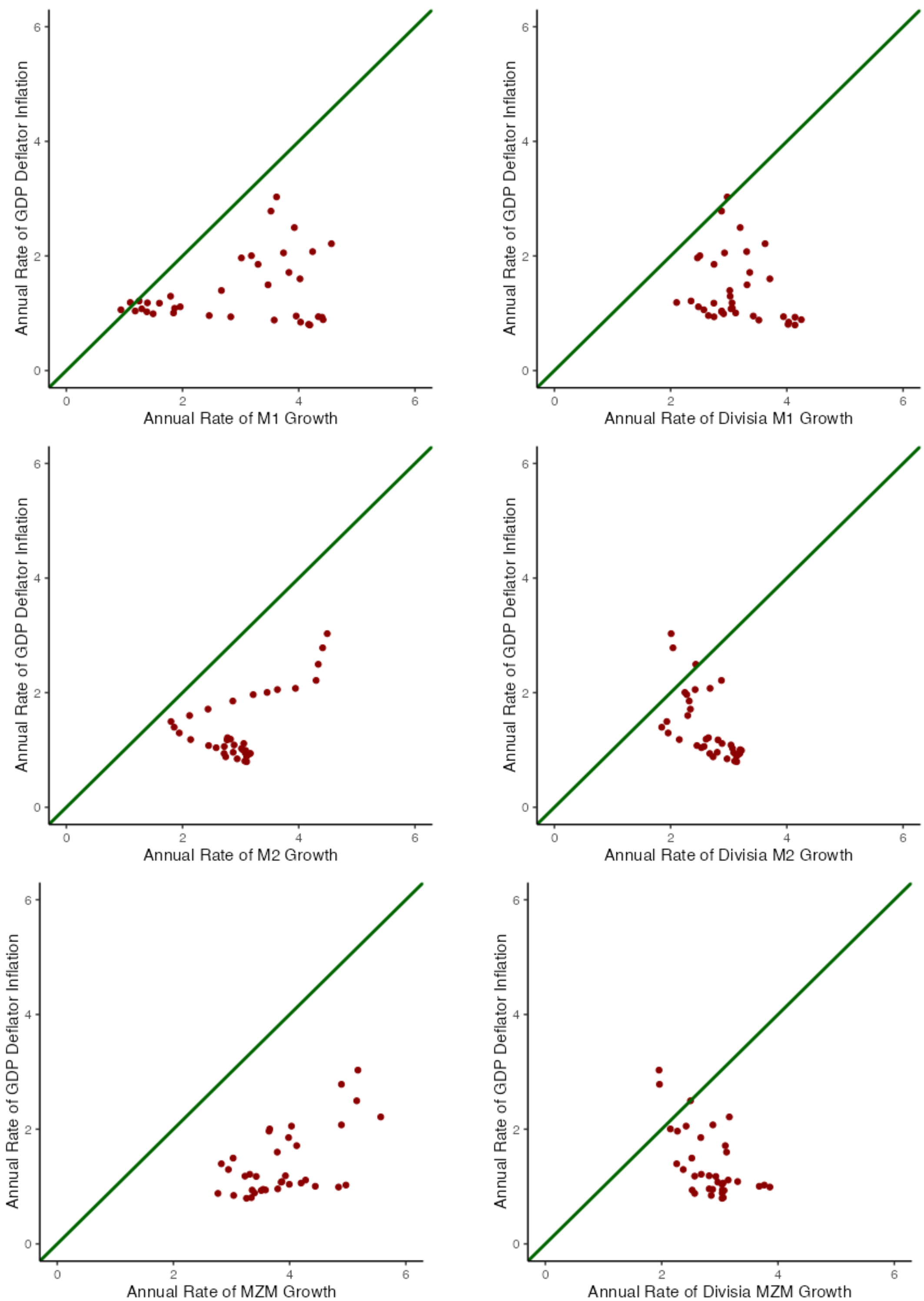
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 33: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.5$); 1984–2019



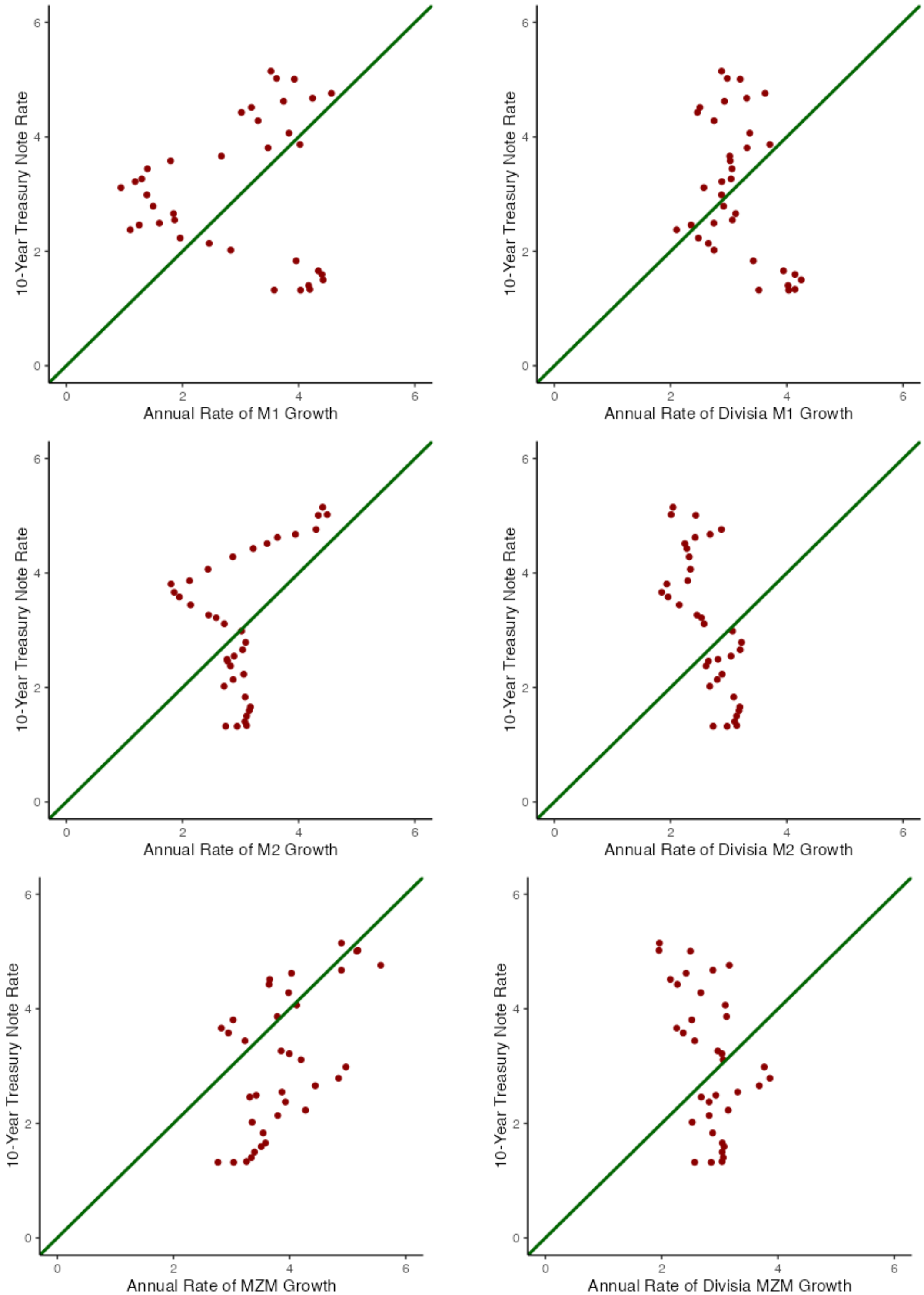
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 34: Scatter Plots of EWMA Filtered Inflation and the Money Growth ($\beta = 0.95$); 1984–2019



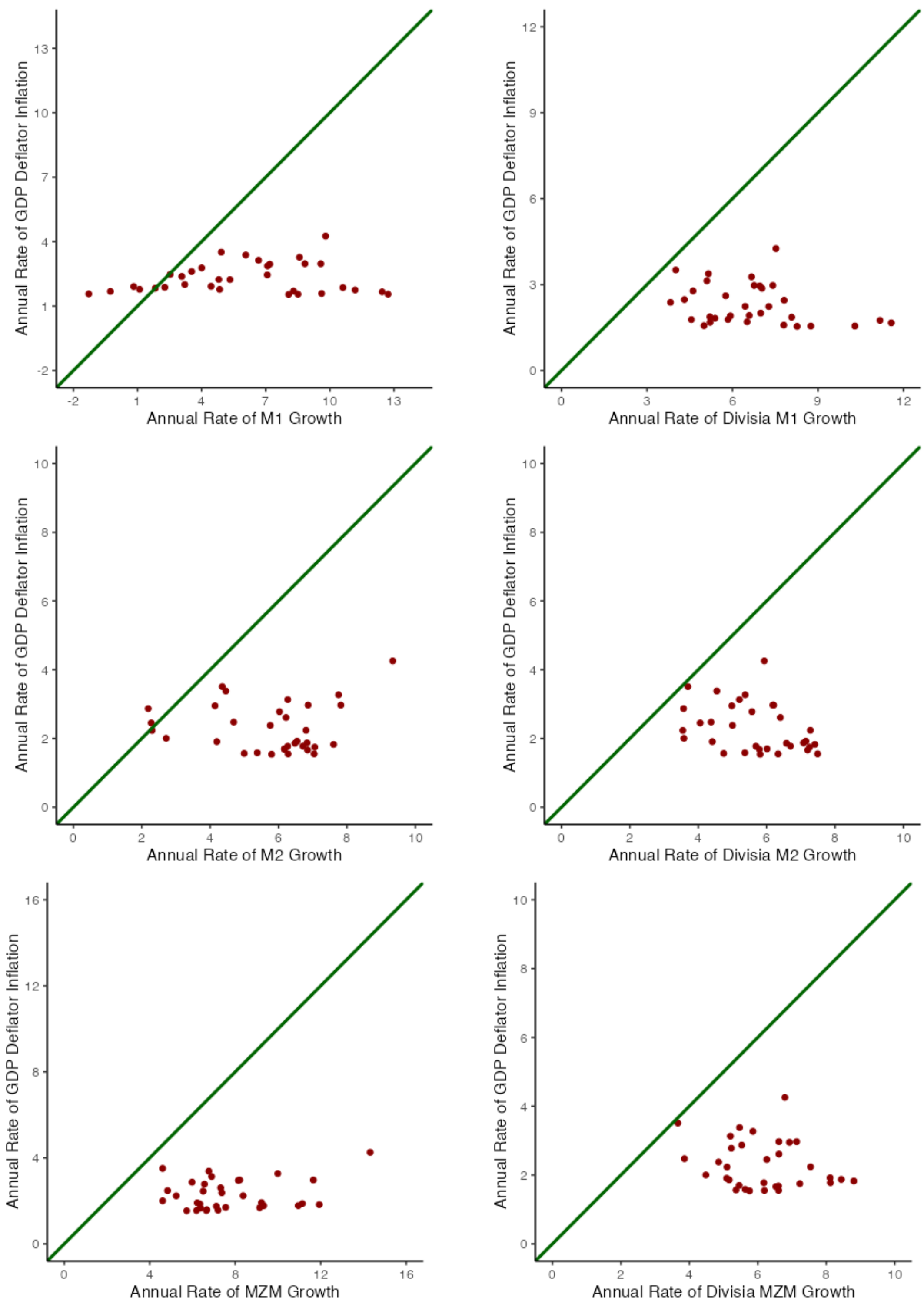
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 35: Scatter Plots of EWMA Filtered Long-term Interest Rate and the Money Growth ($\beta = 0.95$); 1984–2019



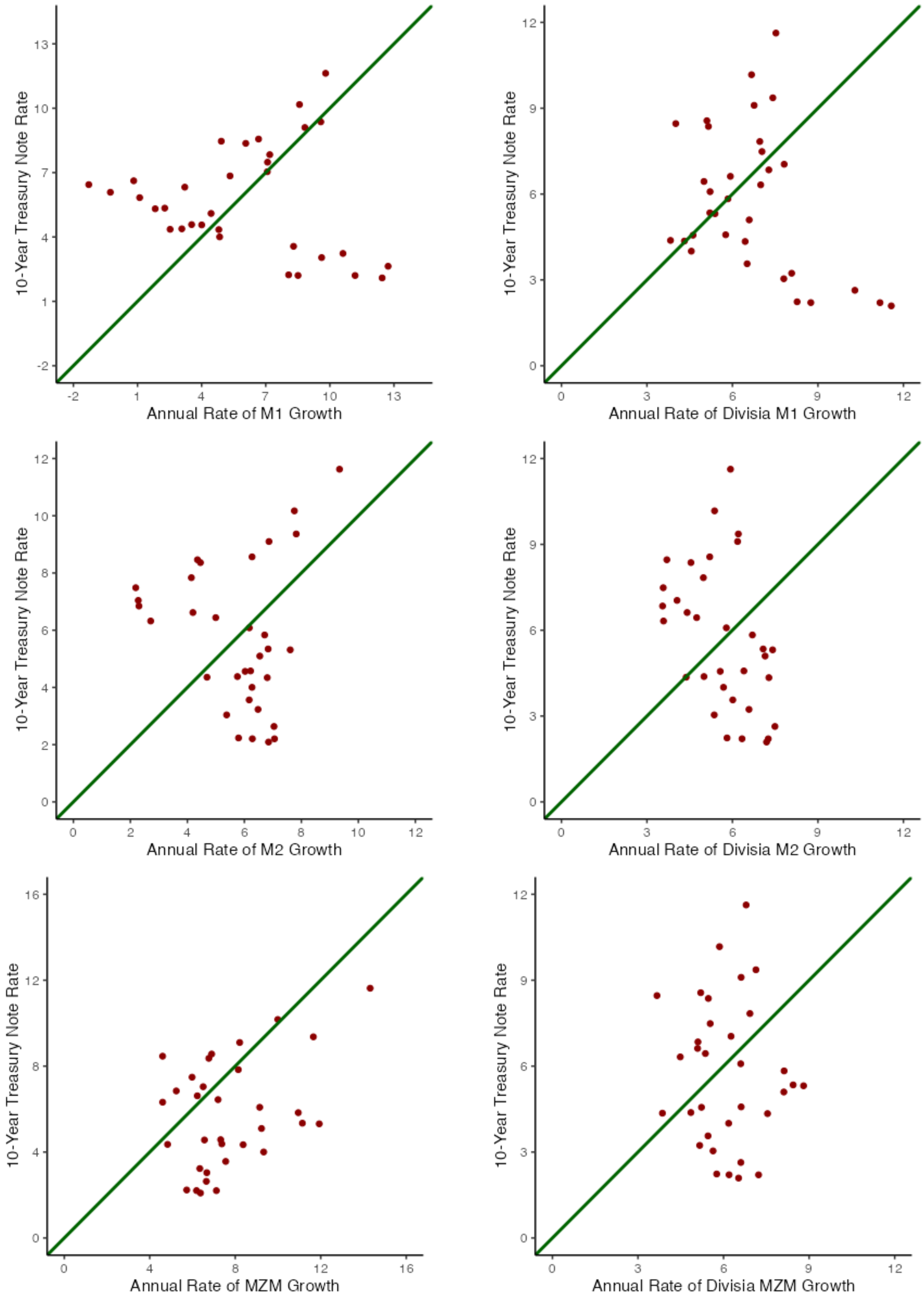
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 36: Scatter Plots of BK Filtered Inflation and the Money Growth (min(6) & max(32)); 1984–2019



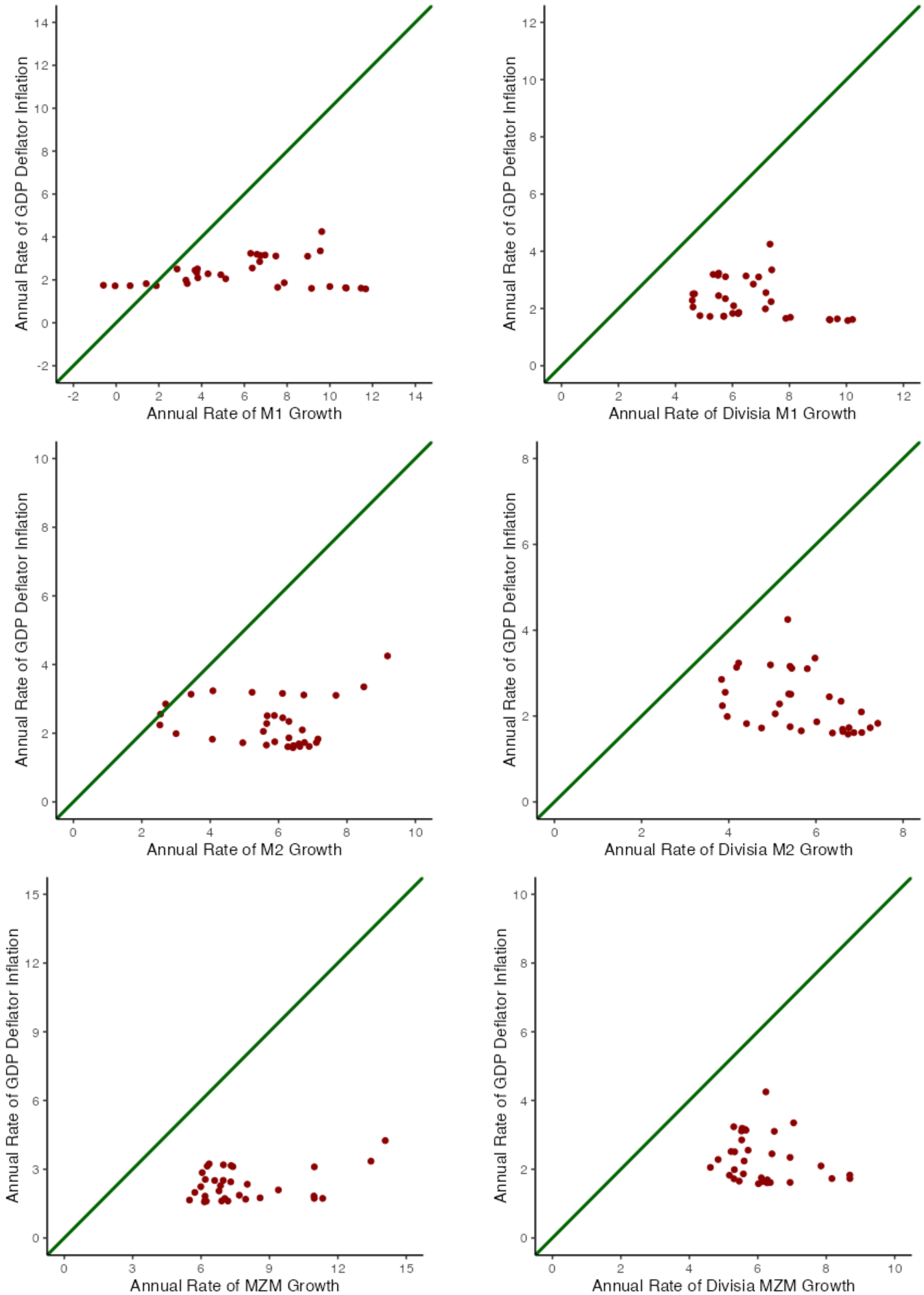
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 37: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(6) & max(32)); 1984–2019



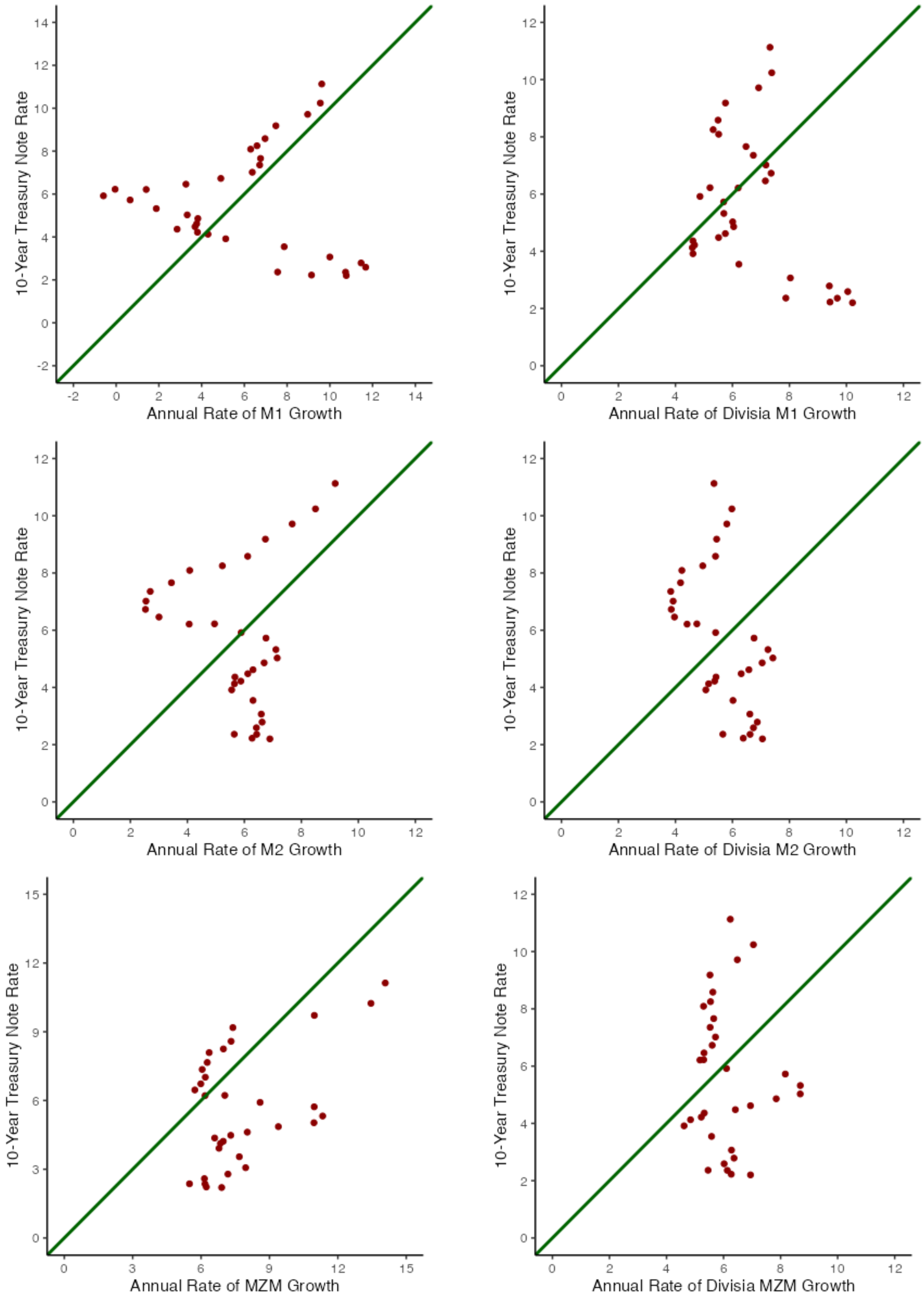
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 38: Scatter Plots of BK Filtered Inflation and the Money Growth (min(2) & max(60)); 1984–2019



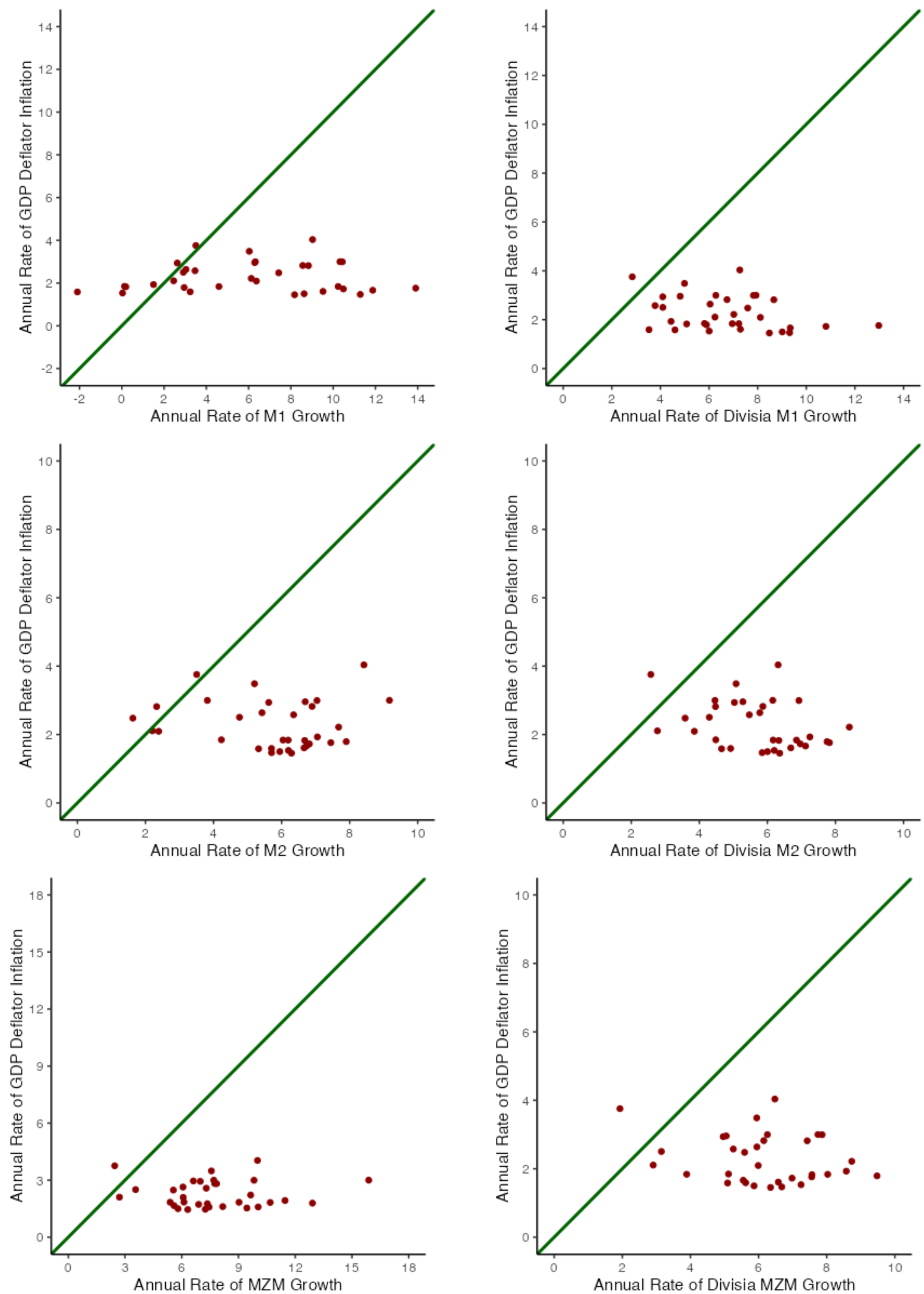
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 39: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(2) & max(60)); 1984–2019



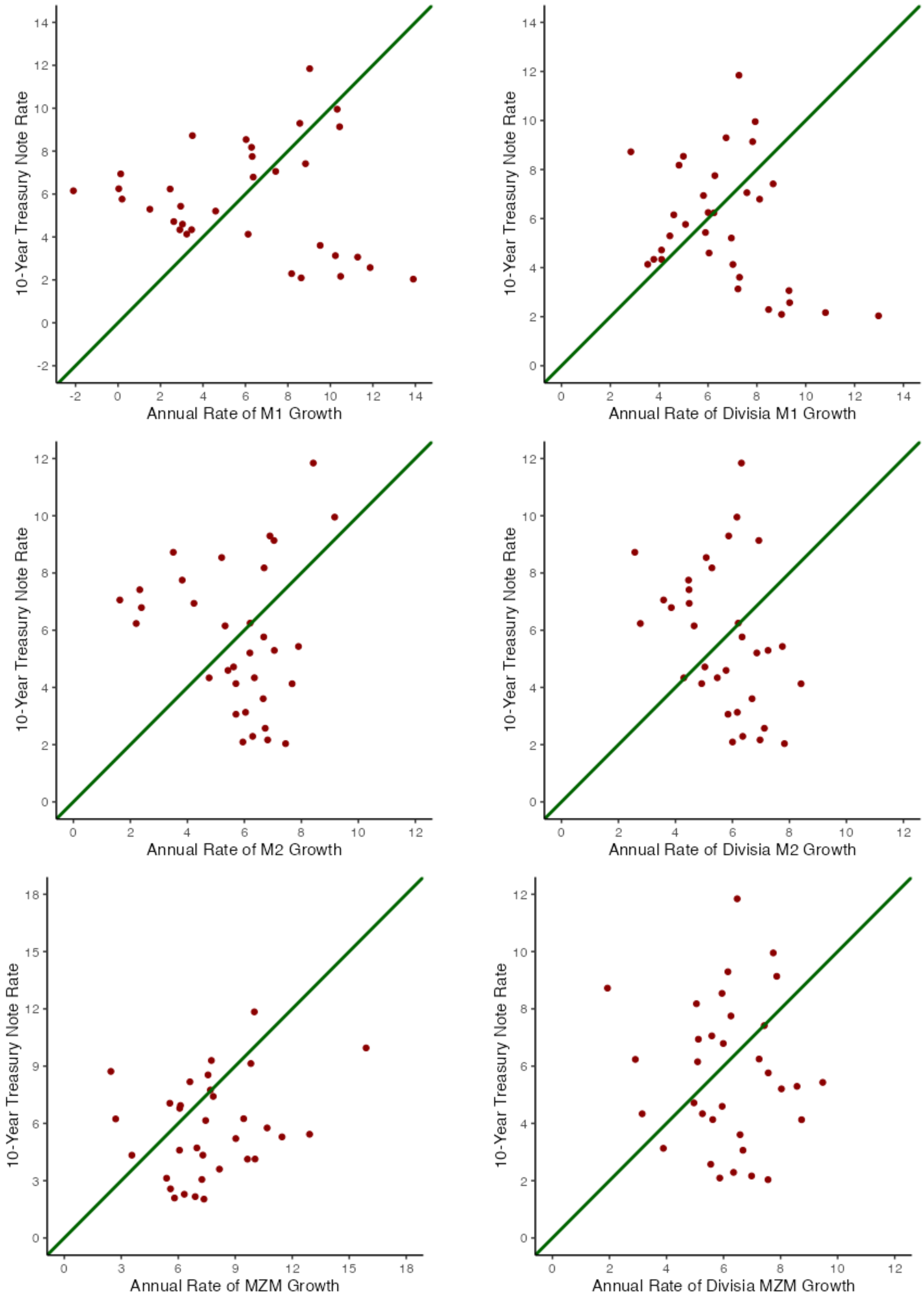
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 40: Scatter Plots of BK Filtered Inflation and the Money Growth (min(8) & max(30)); 1984–2019



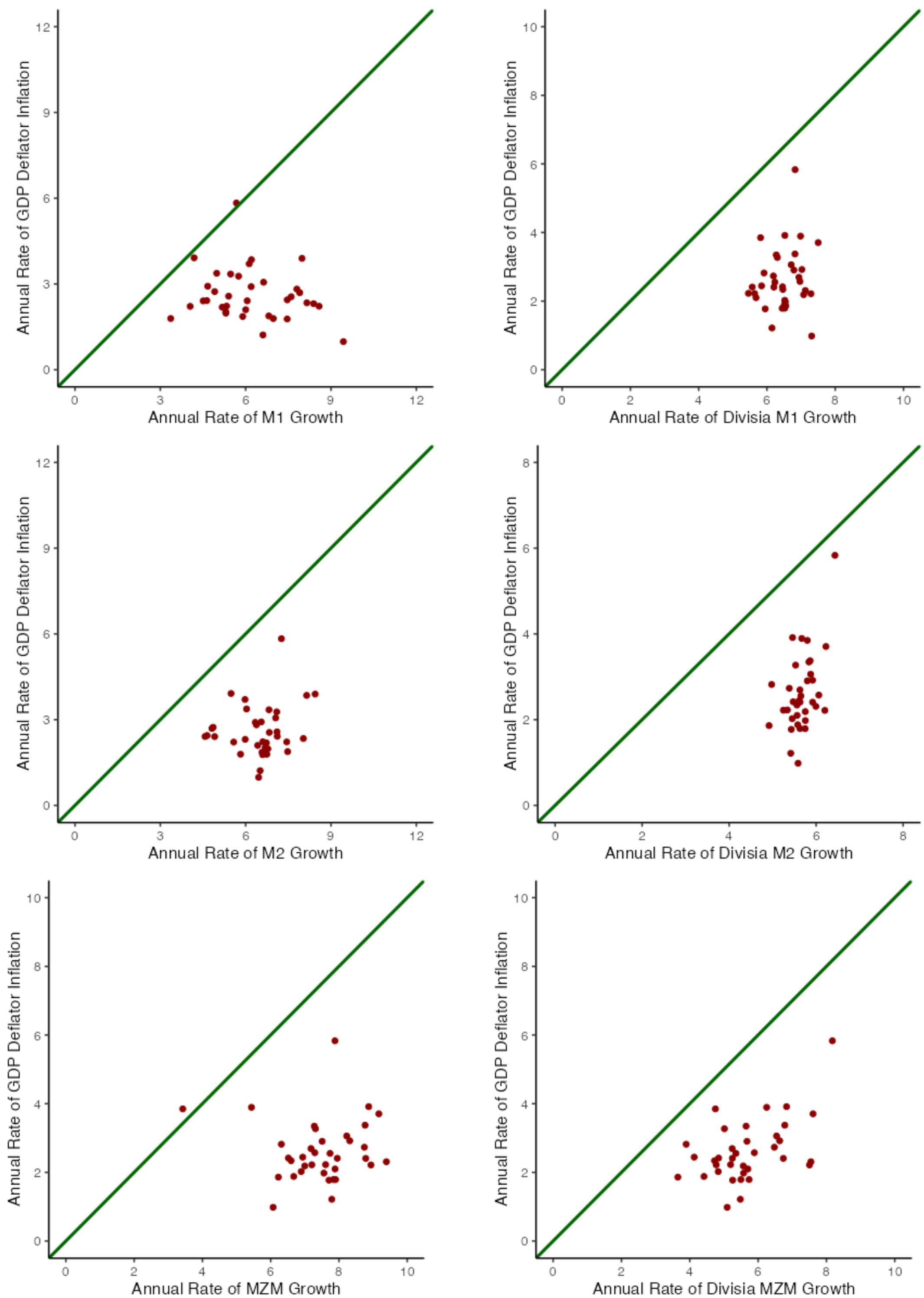
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 41: Scatter Plots of BK Filtered Long-term Interest Rate and the Money Growth (min(8) & max(30)); 1984–2019



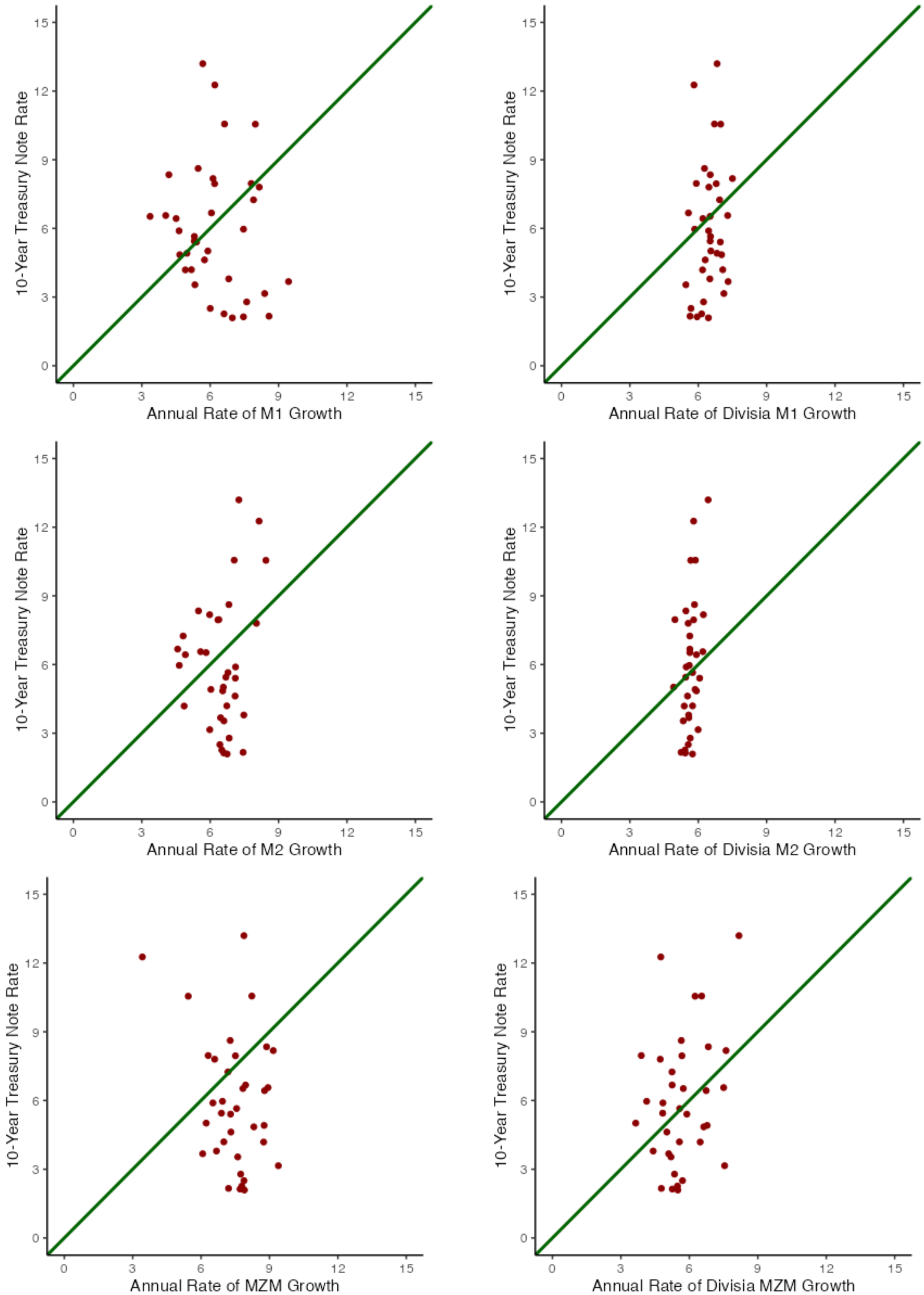
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 42: Scatter Plots of Hamilton Filtered Inflation and the Money Growth; 1984–2019



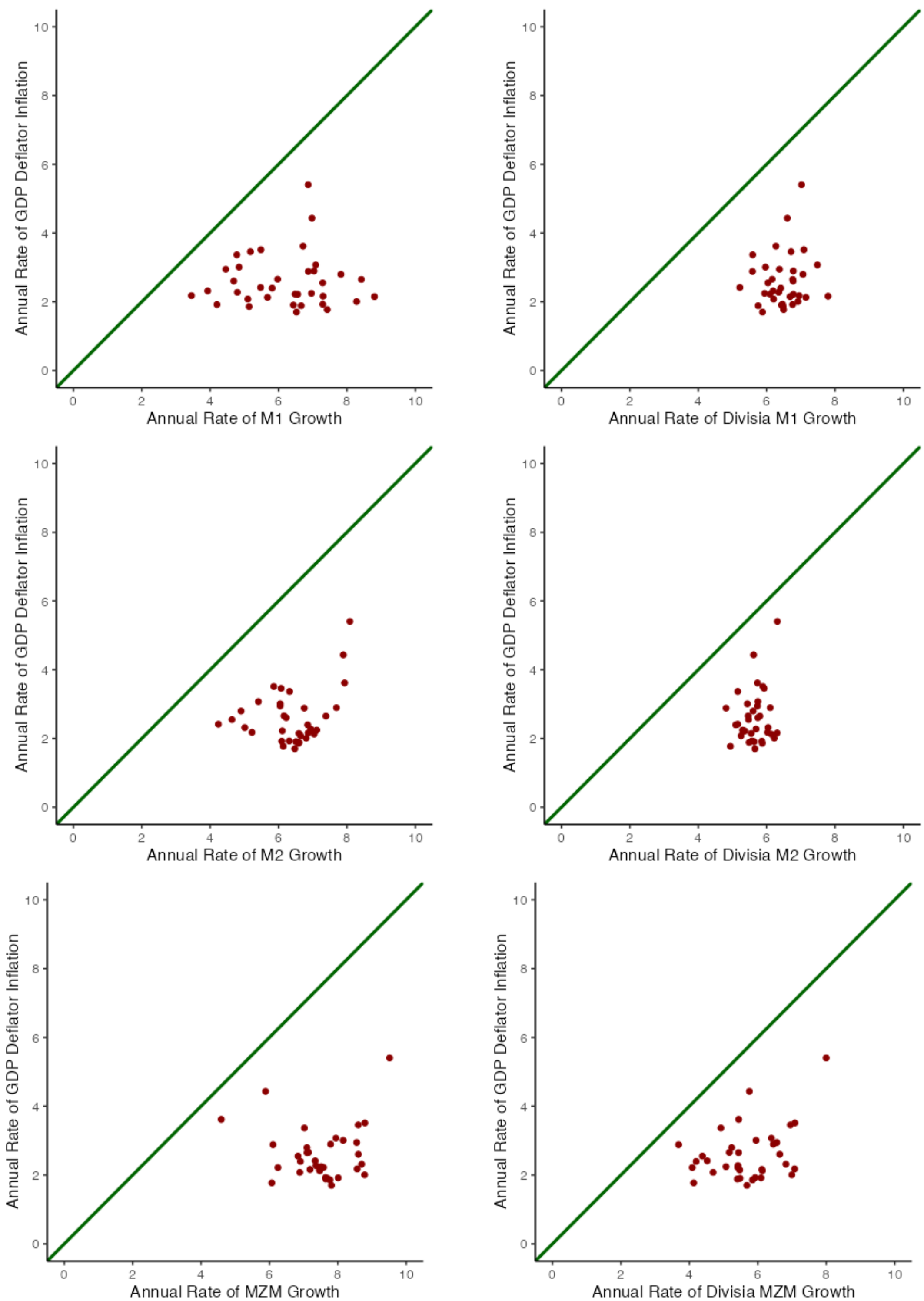
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 43: Scatter Plots of Hamilton Filtered Long-term Interest Rate and the Money Growth; 1984–2019



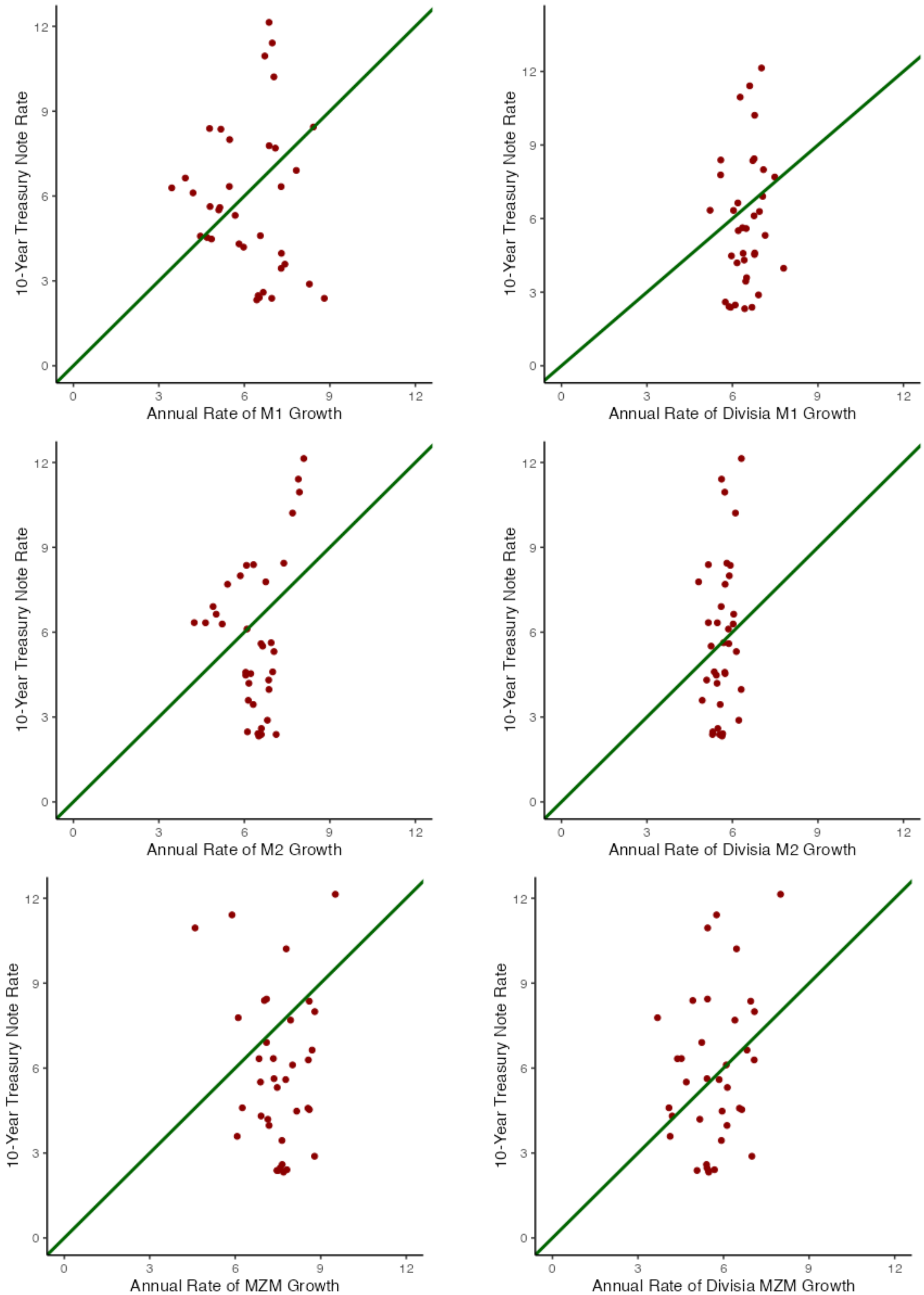
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 44: Scatter Plots of Modified Hamilton Filtered Inflation and the Money Growth; 1984–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure 45: Scatter Plots of Modified Hamilton Filtered Long-term Interest Rate and the Money Growth; 1984–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

4 Concluding Remarks

I follow the steps of previous studies that have re-examined the relationships proposed by Lucas regarding the quantity theory of money. The analysis involved different measures of the money supply and various filtering techniques.

The findings showed that no significant relationship was found between money growth and inflation or interest rates in the full sample analysis without filtering. This suggested the effectiveness of counter-cyclical policies in controlling inflation and independently influencing interest rates.

When filtering techniques were applied, patterns emerged. The exponential weighted moving average (EWMA) and Baxter-King (BK) filters demonstrated that increasing the smoothing parameter or widening the window range improved alignment with quantity-theoretic predictions, particularly for simple-sum aggregates.

The Hamilton filter results favored the Divisia aggregates over simple-sum aggregates in conforming to quantity-theoretic predictions. However, the modified Hamilton filter showed better alignment for Divisia and simple-sum MZM aggregates.

Examining two subsamples reinforced the influence of filtering techniques. The EWMA and BK filters produced closer alignment with quantity-theoretic predictions, especially for simple-sum M1 and Divisia M1 aggregates in the first subsample.

In the second subsample, the BK filter did not consistently exhibit quantity-theoretic predictions. However, when the EWMA filter is applied, positive relationships between money growth and inflation and money growth and the interest rates were observed primarily for the simple-sum aggregates. In contrast, for the Hamilton and the modified Hamilton filters, such relationships are observed mainly for the Divisia aggregates.

What can we make from the above results? First, money supply measures and filtering techniques are important when analyzing the relationships between money growth, inflation, and interest rates. Second, the Divisia M2 and MZM seem to perform well in every sample using the Hamilton filter. Finally, it would be useful to apply the alternative measure of money supply and filtering techniques to other countries, including the less developed ones, and see whether any broad patterns emerge.

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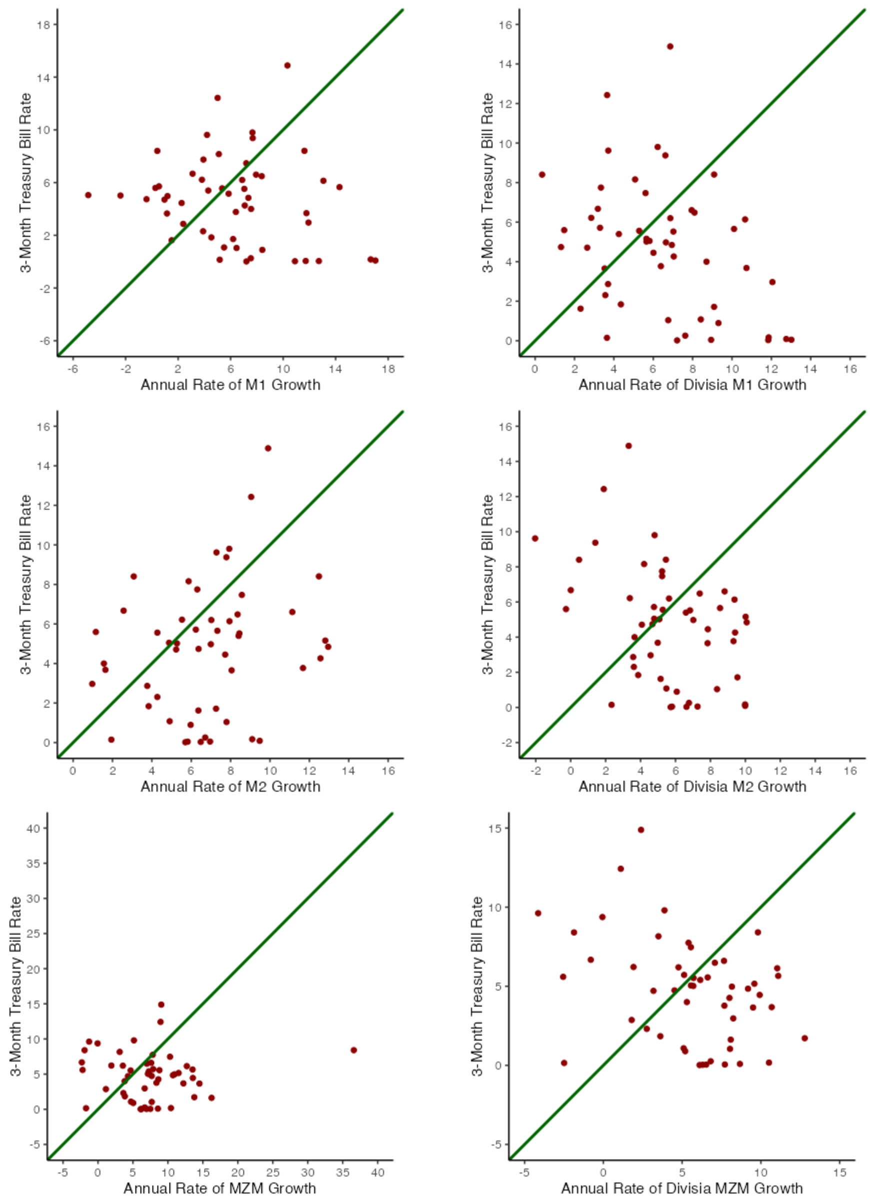
Appendix

A The Full Sample: Original Data for 1968 – 2019

A.1 Original Data without Filter

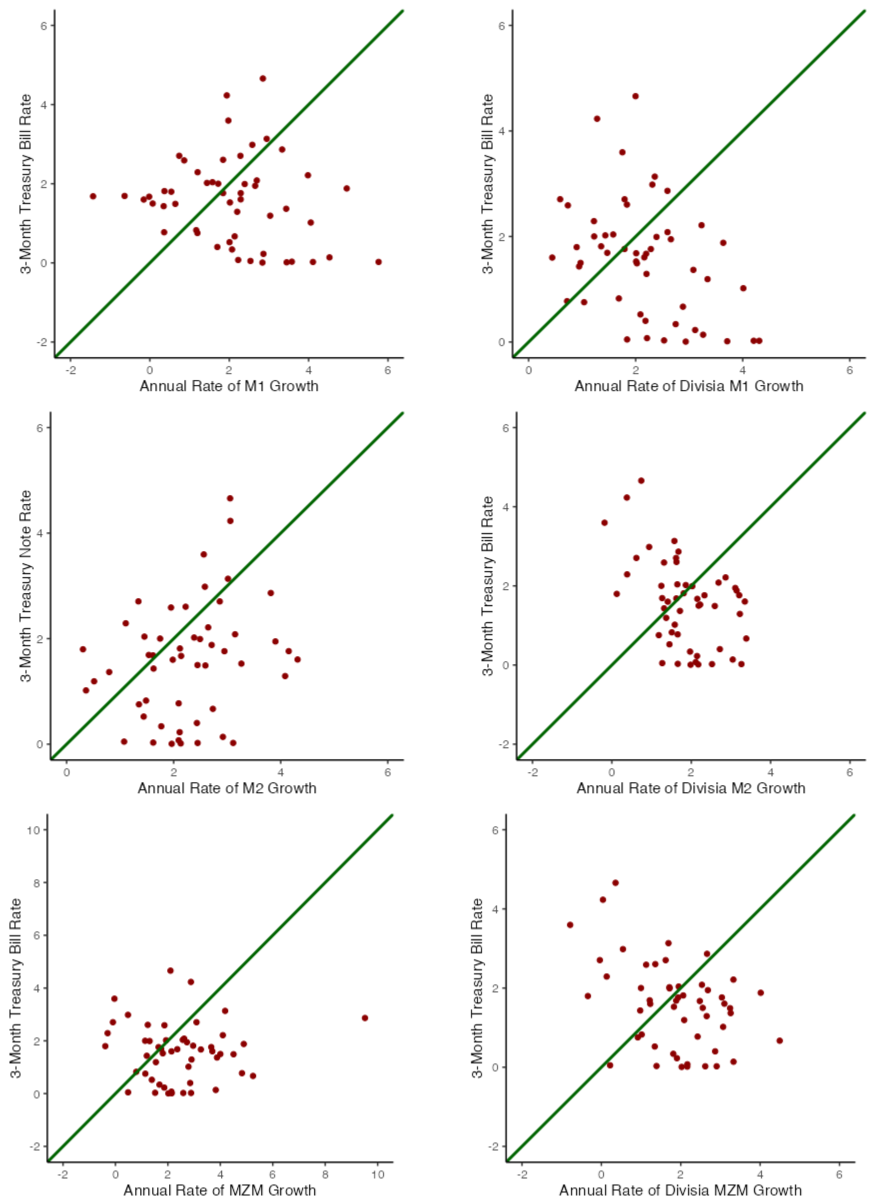
Figure A.1: Scatter Plots of Unfiltered Short-term Interest Rate and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

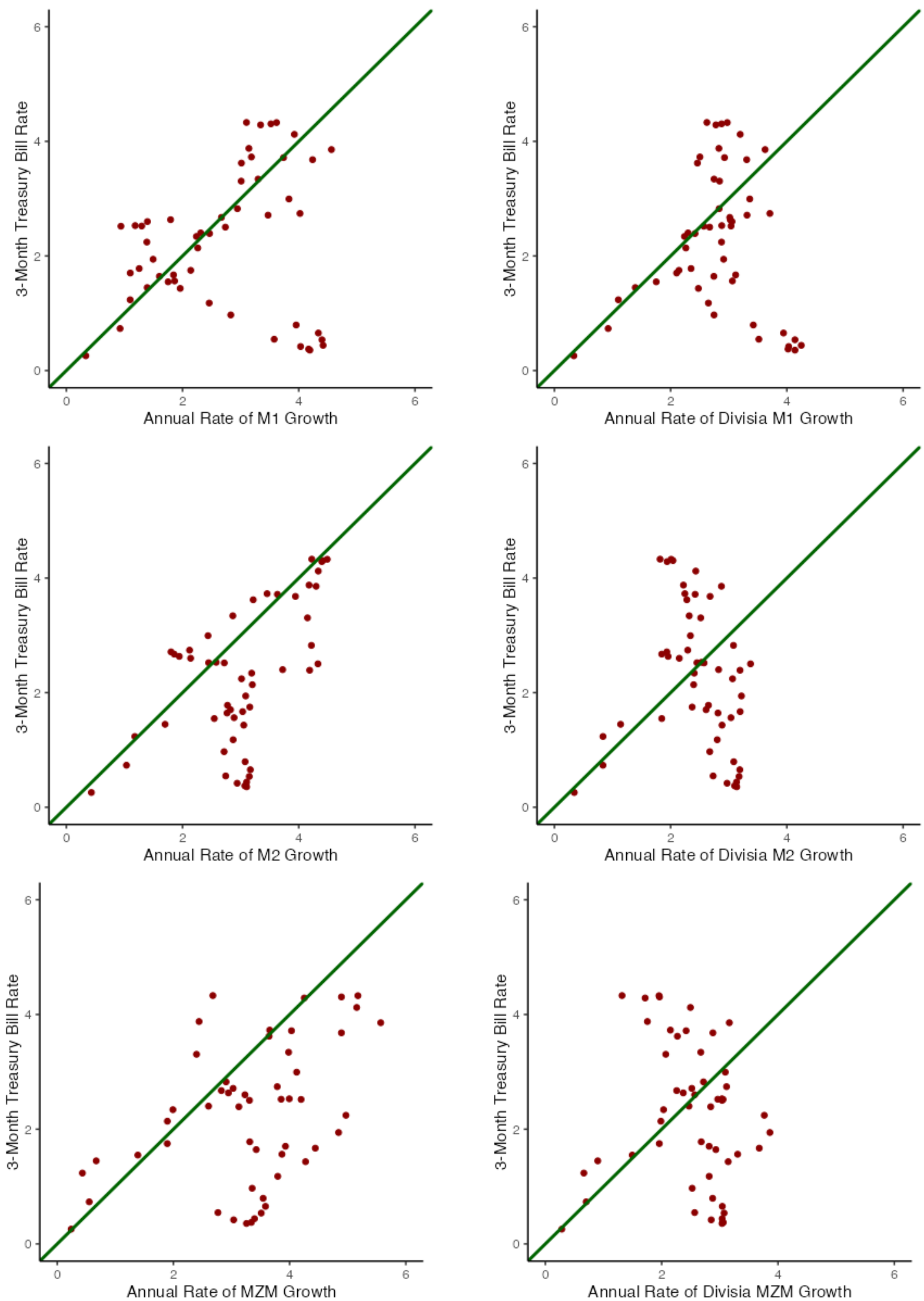
A.2 Smoothed Data

Figure A.2: Scatter Plots of EWMA Filtered Short-term Interest Rate and the Money Growth ($\beta = 0.5$); 1968–2019



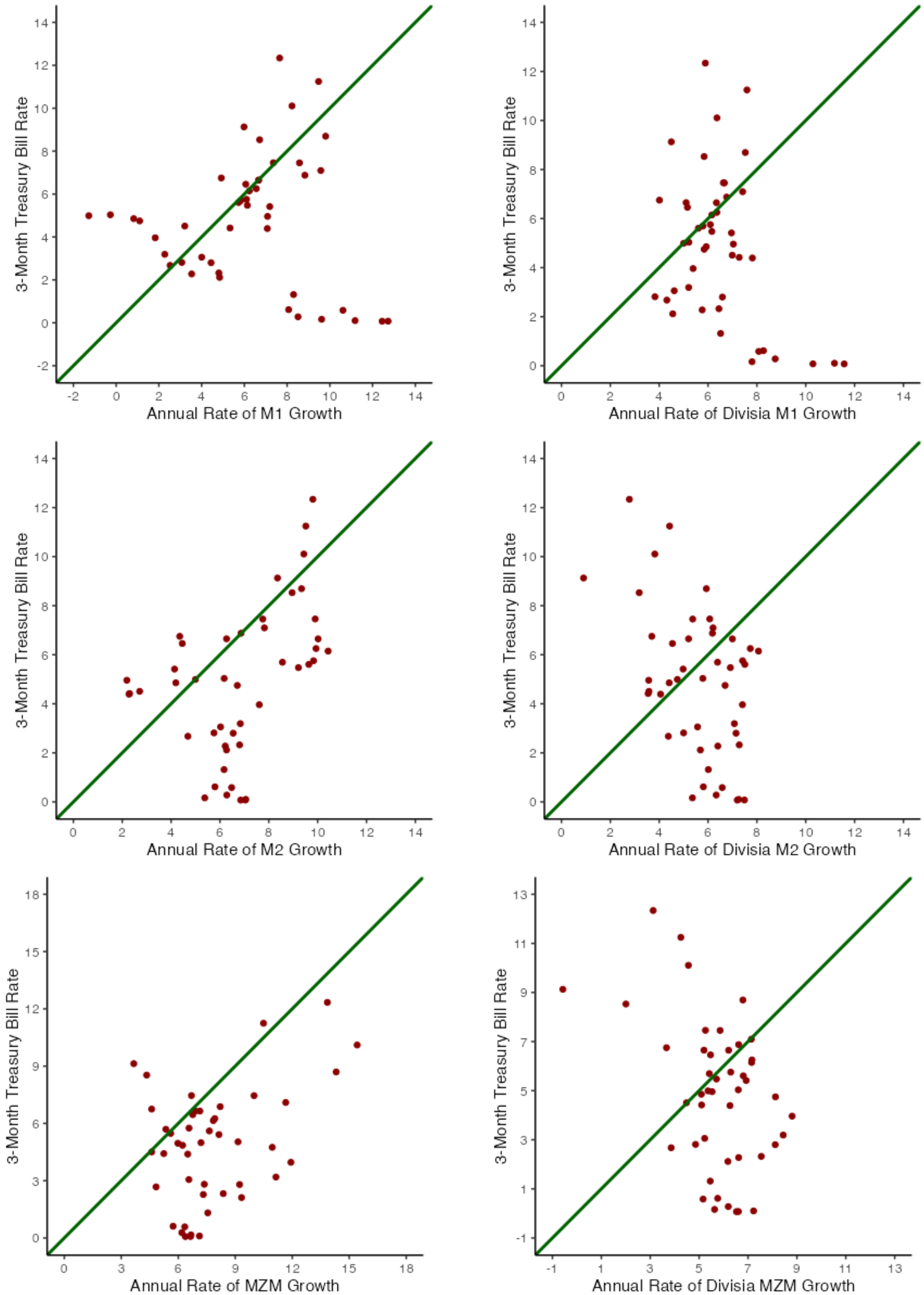
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.3: Scatter Plots of EWMA Filtered Short-term Interest Rate and the Money Growth ($\beta = 0.95$); 1968–2019



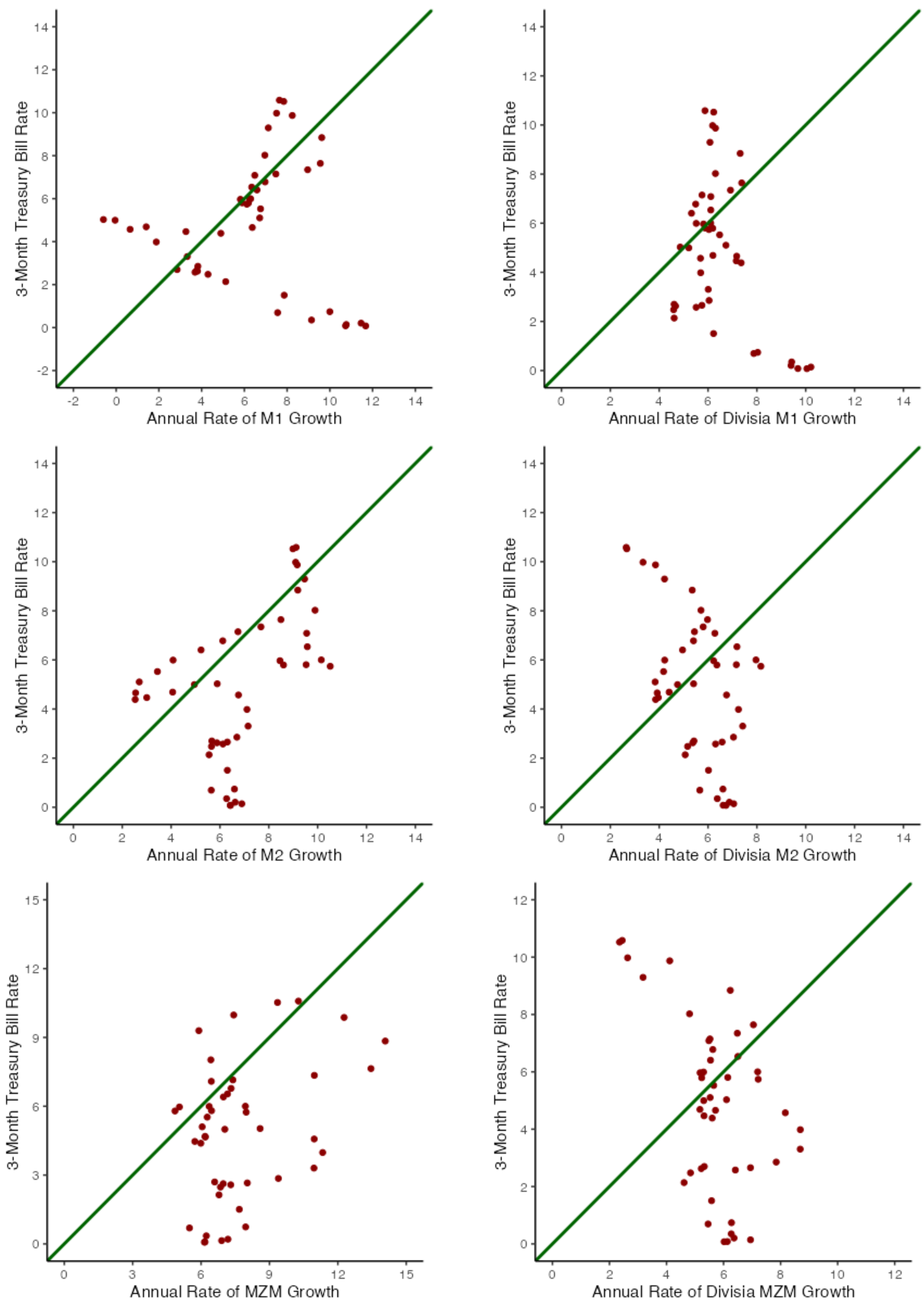
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.4: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(6) & max(32)); 1968–2019



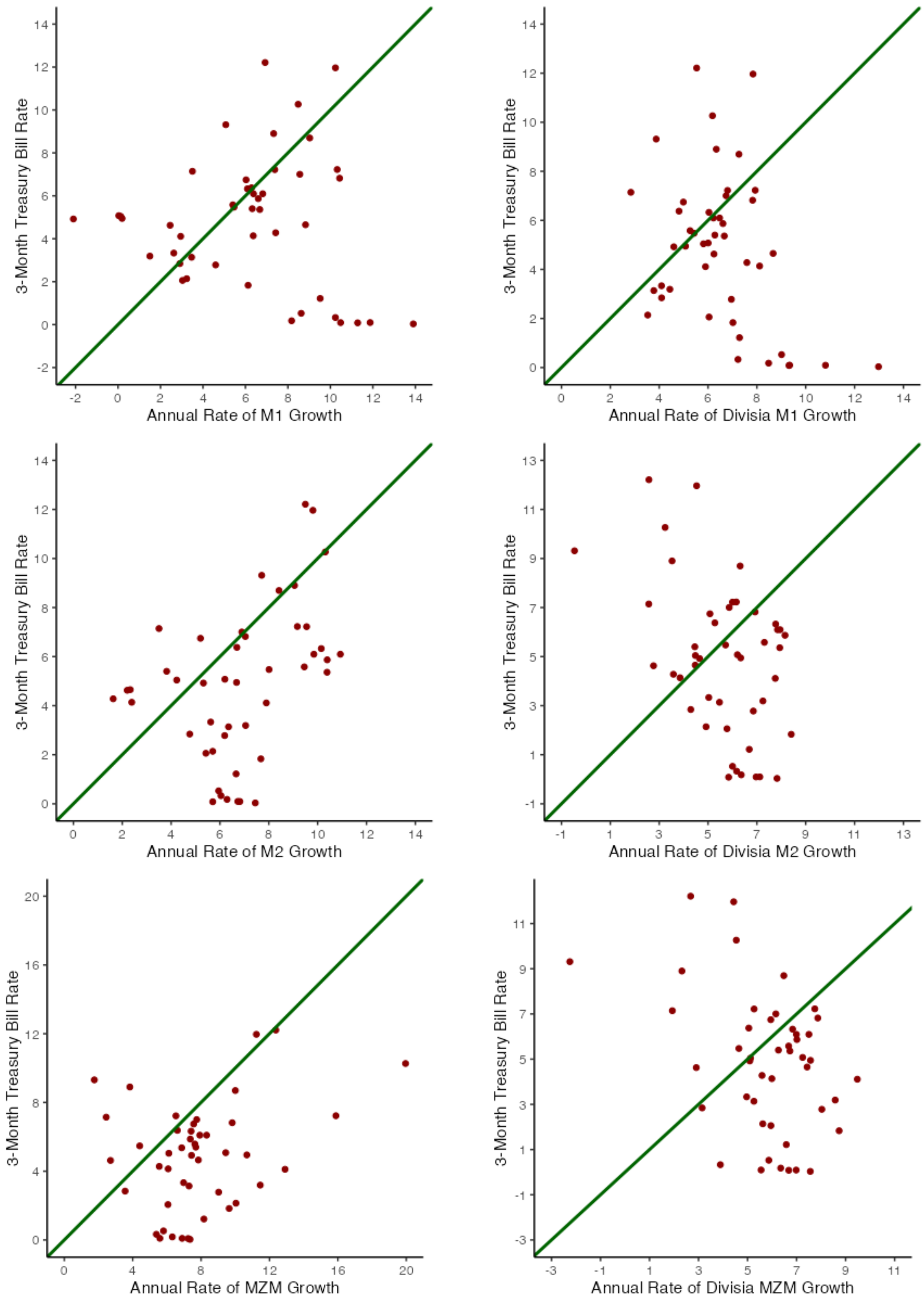
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.5: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(2) & max(60)); 1968–2019



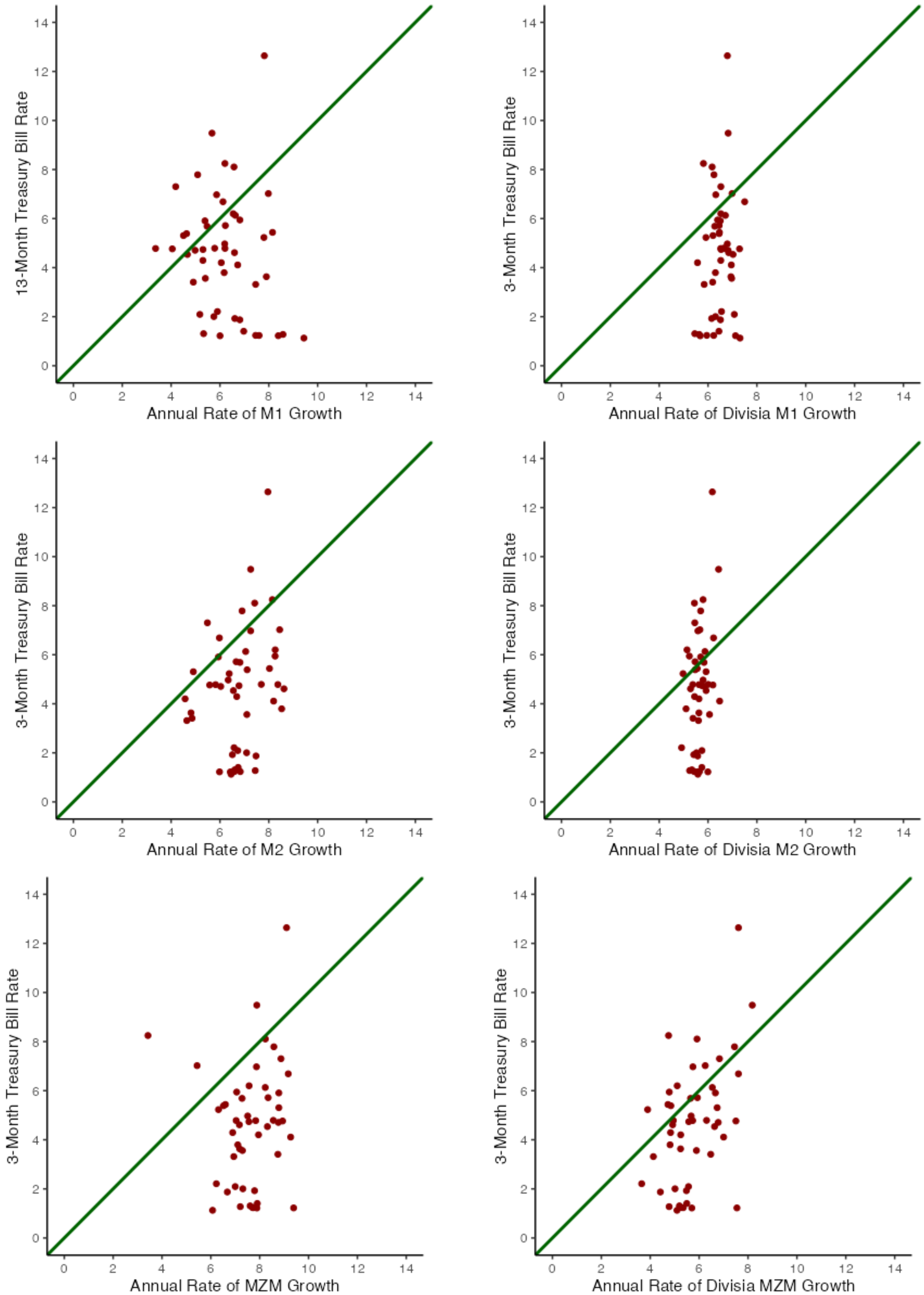
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.6: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(8) & max(30)); 1968–2019



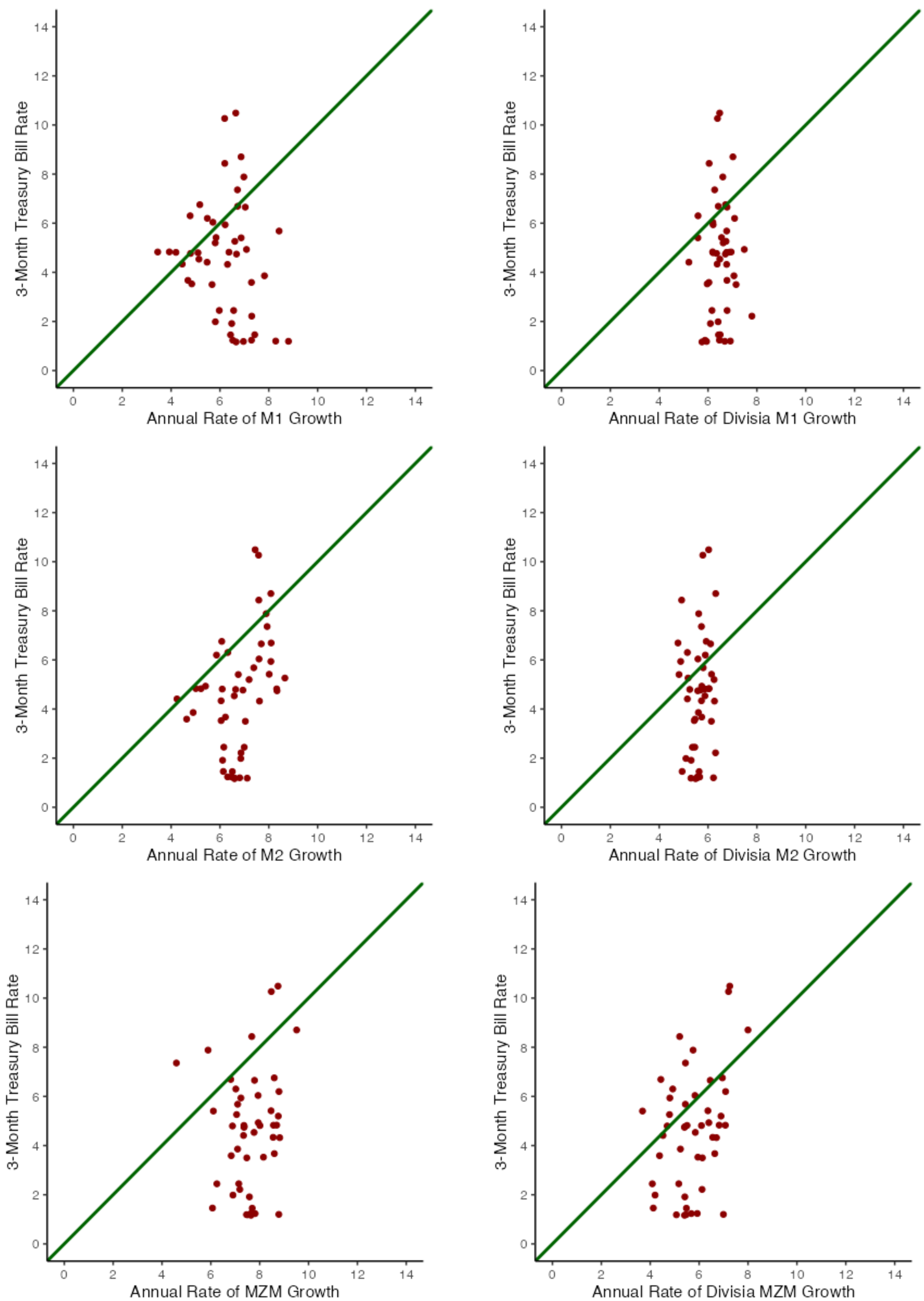
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.7: Scatter Plots of Hamilton Filtered Short-term Interest Rate and the Money Growth; 1968–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.8: Scatter Plots of Modified Hamilton Filtered Short-term Interest Rate and the Money Growth; 1968–2019

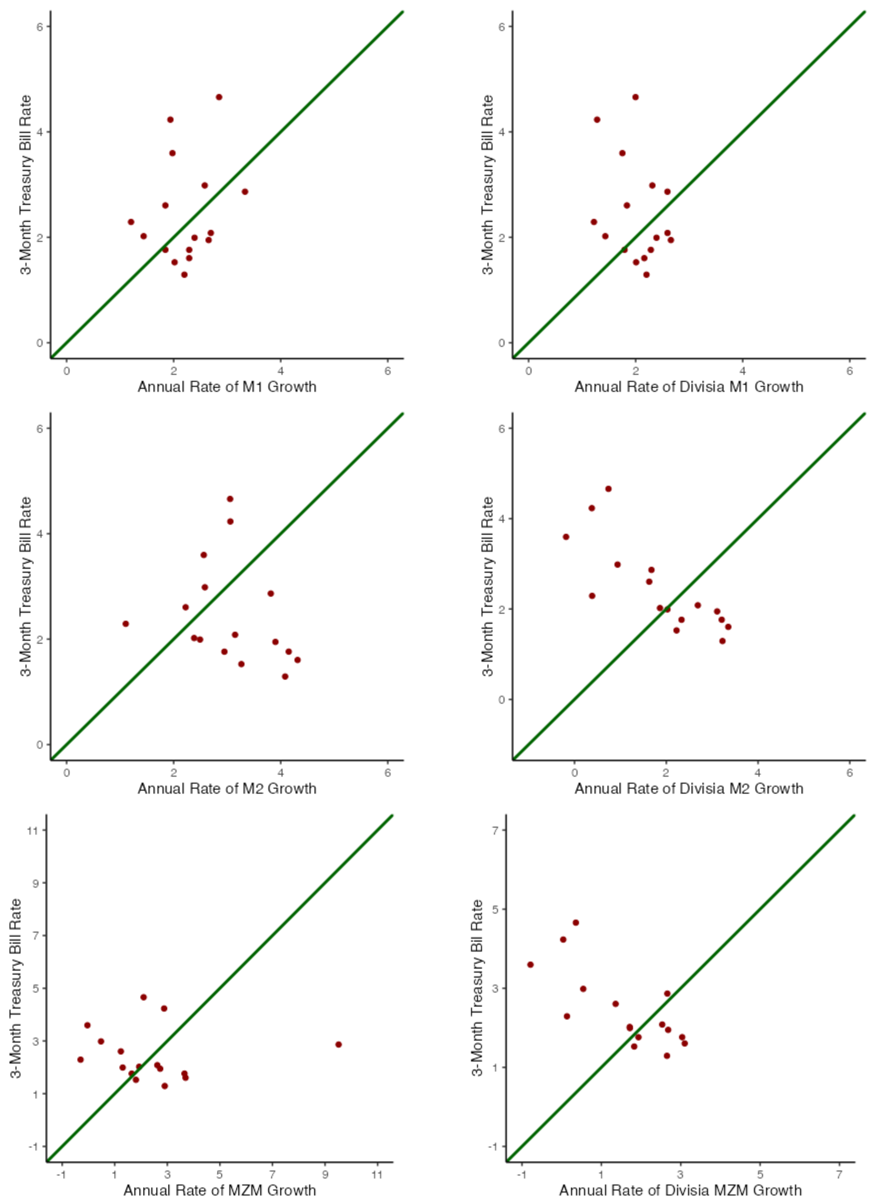


Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

A.3 The Early Subsample: Smoothed Data for 1968–83

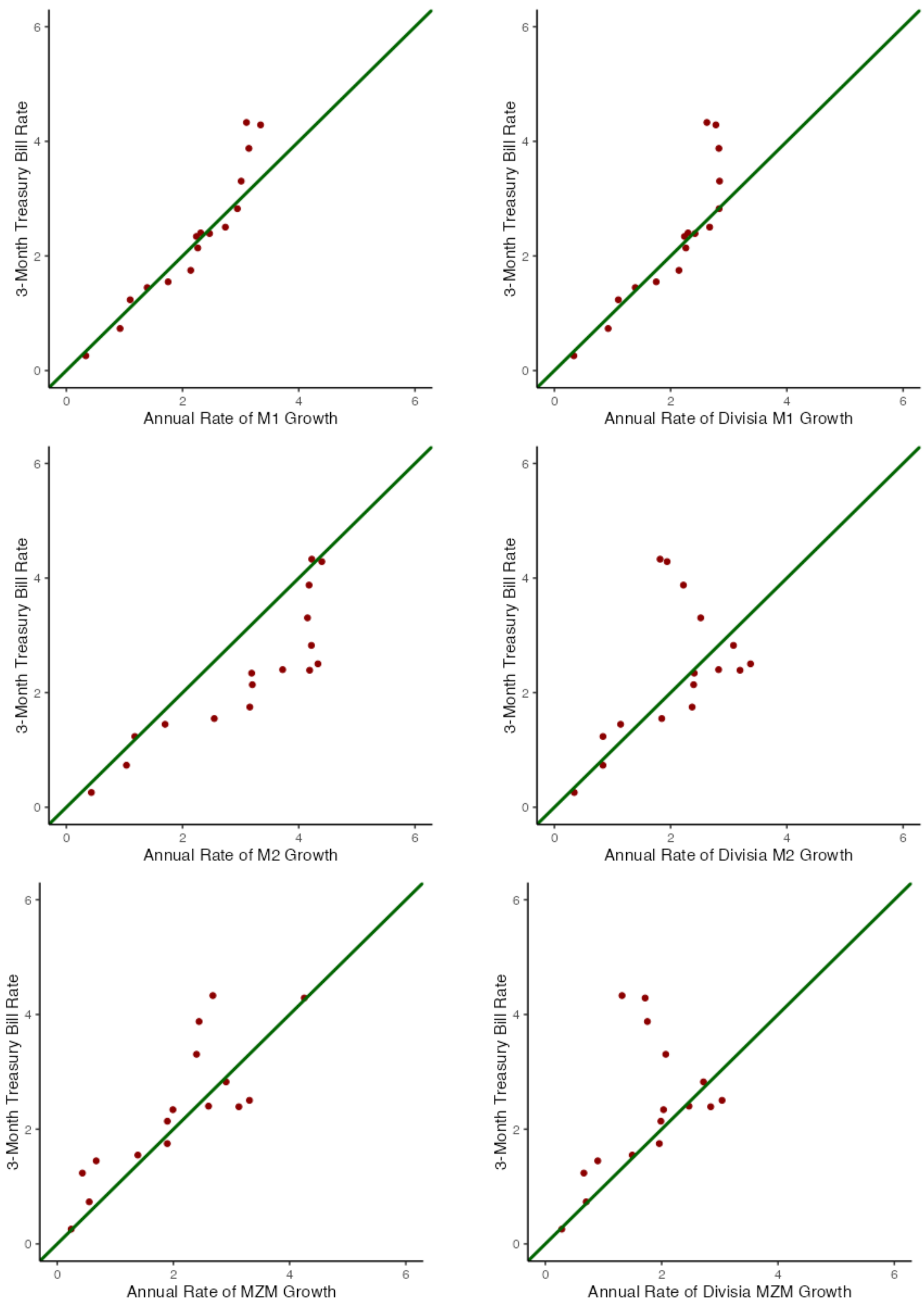
A.3.1 Smoothed Data

Figure A.9: SCATTER PLOTS OF EWMA FILTERED SHORT-TERM INTEREST RATE AND THE MONEY GROWTH ($\beta = 0.5$); 1968–83



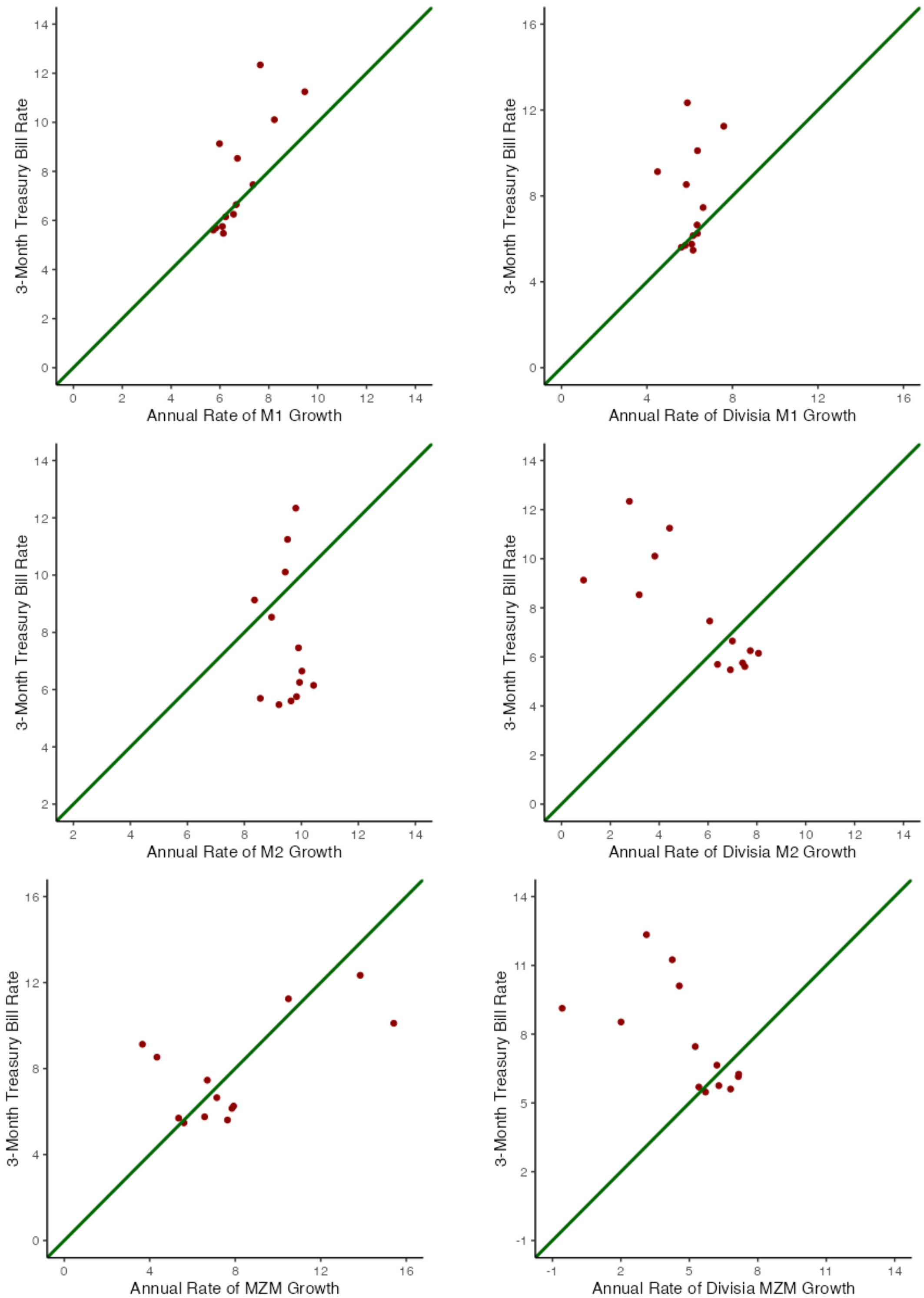
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.10: Scatter Plots of EWMA Filtered Short-term Interest Rate and the Money Growth ($\beta = 0.95$); 1968–83



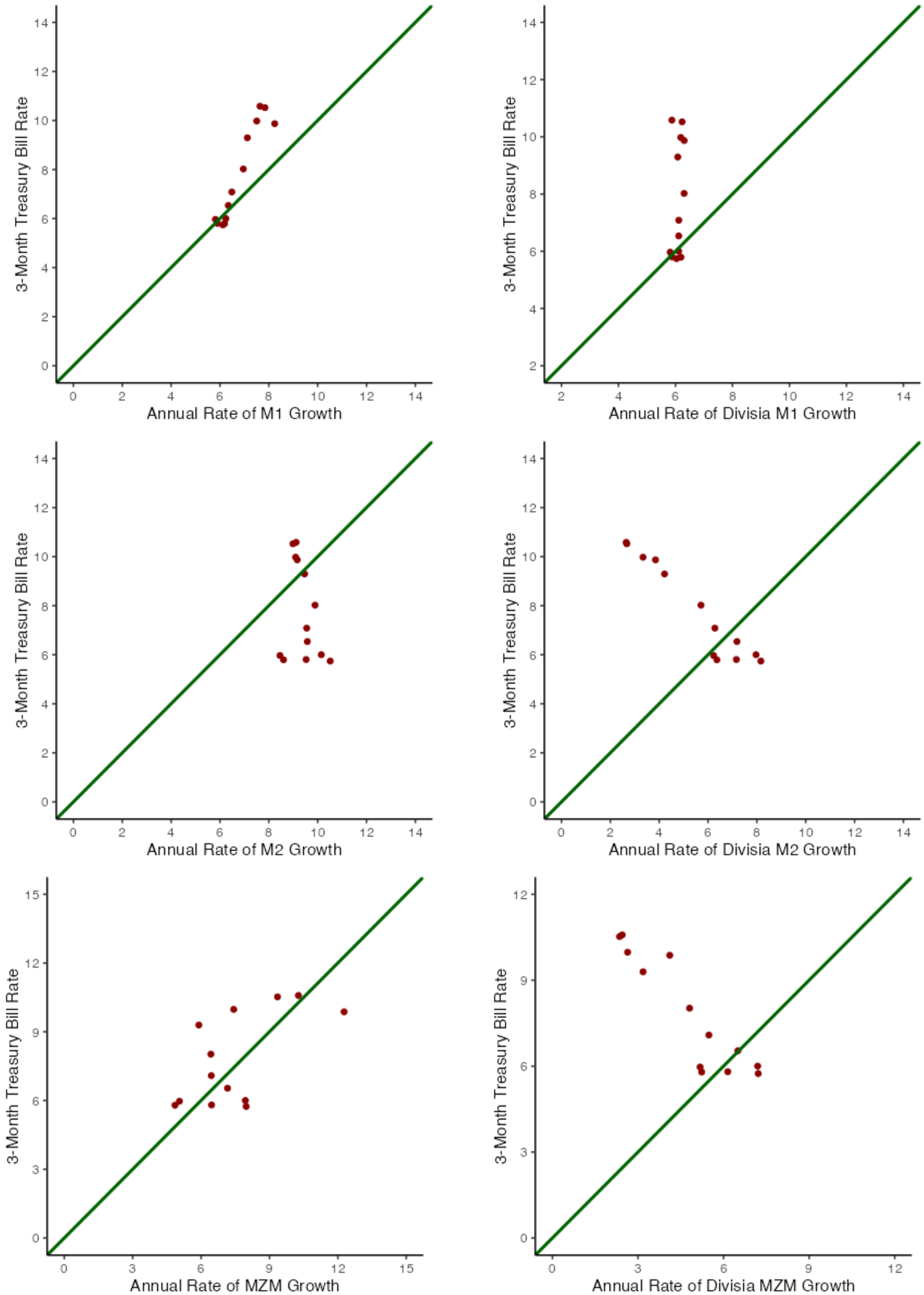
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.11: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(6) & max(32)); 1968–83



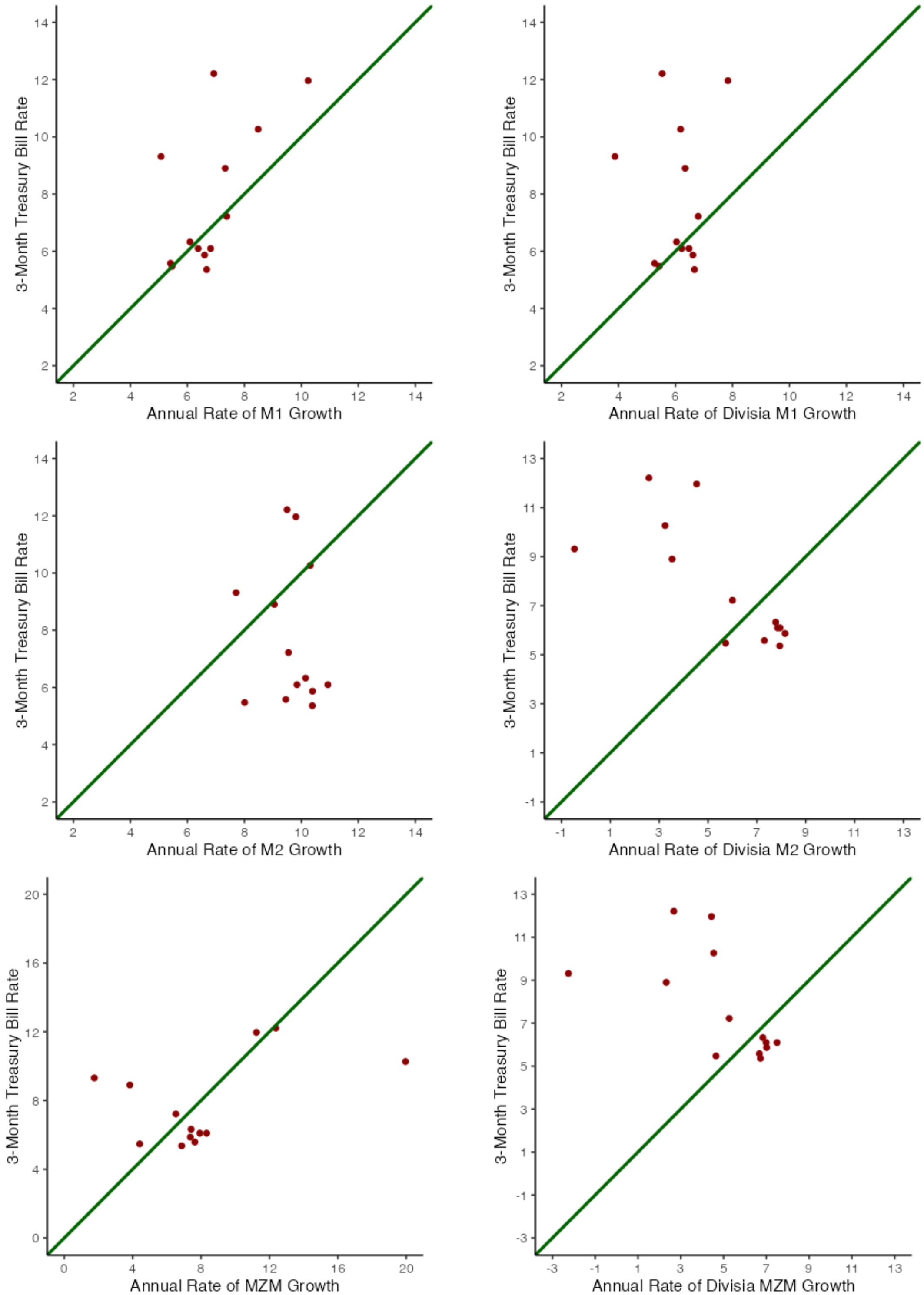
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.12: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(2) & max(60)); 1968–83



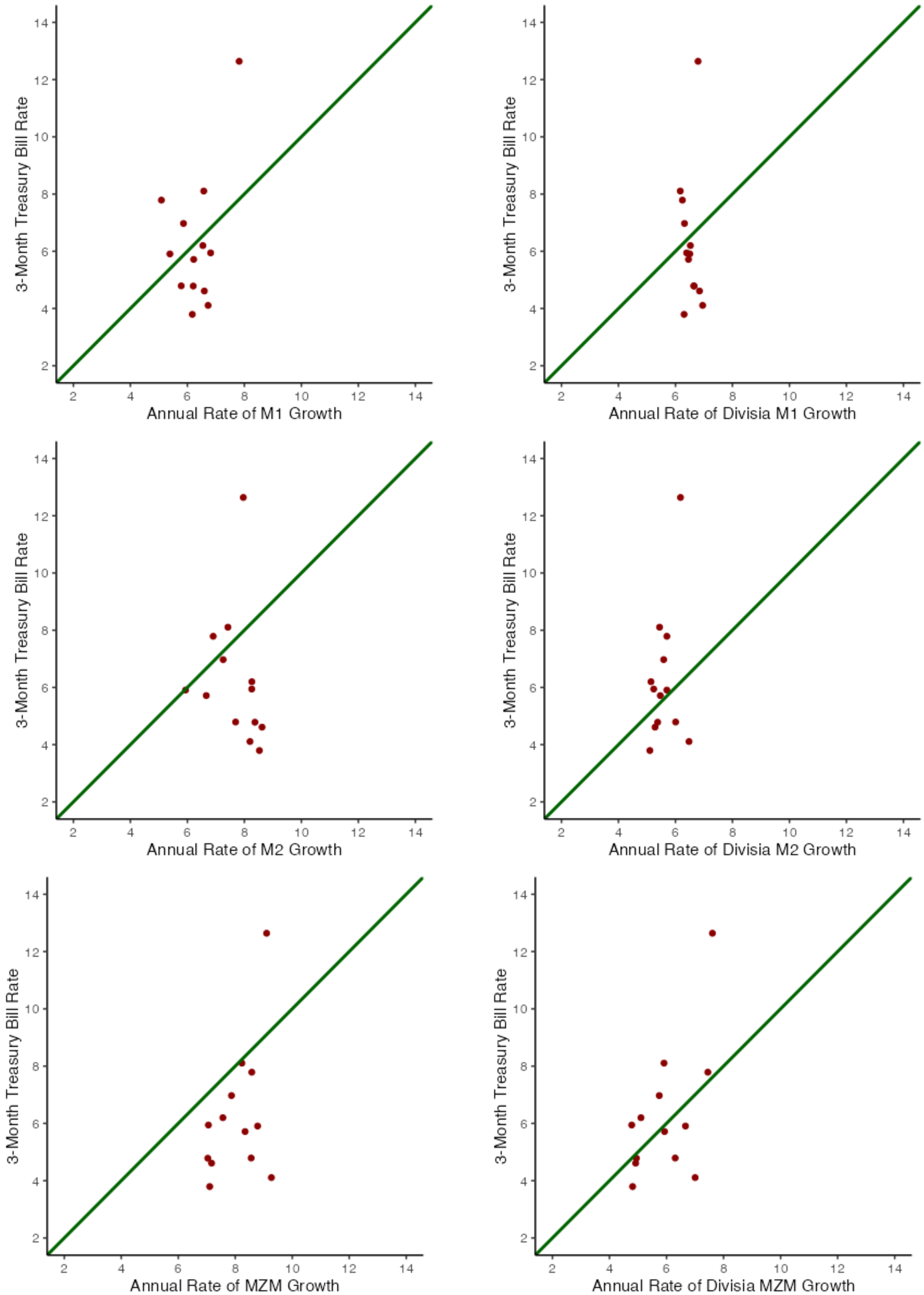
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.13: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(8) & max(30)); 1968–83



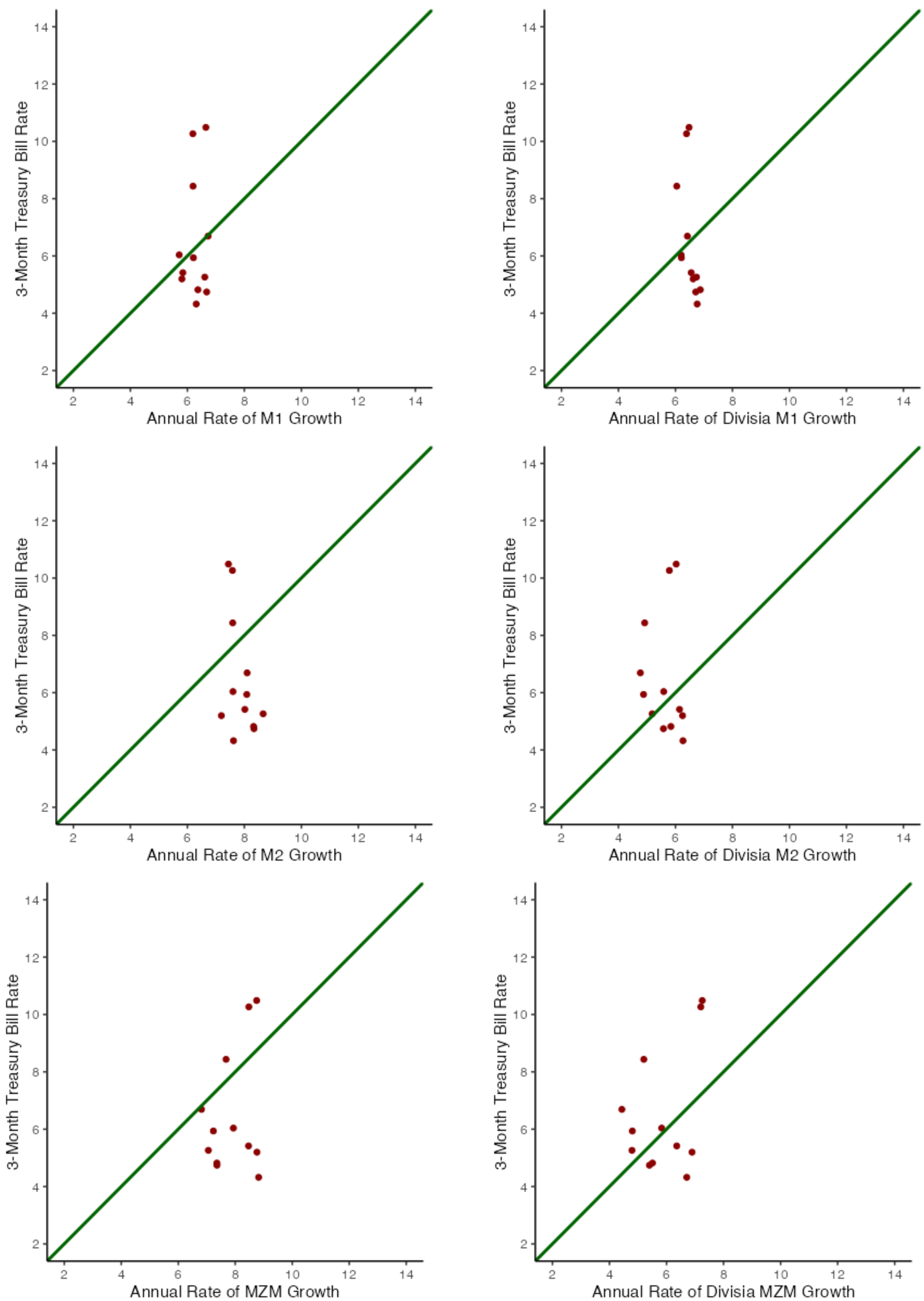
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.14: Scatter Plots of Hamilton Filtered Short-term Interest Rate and the Money Growth; 1968–83



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

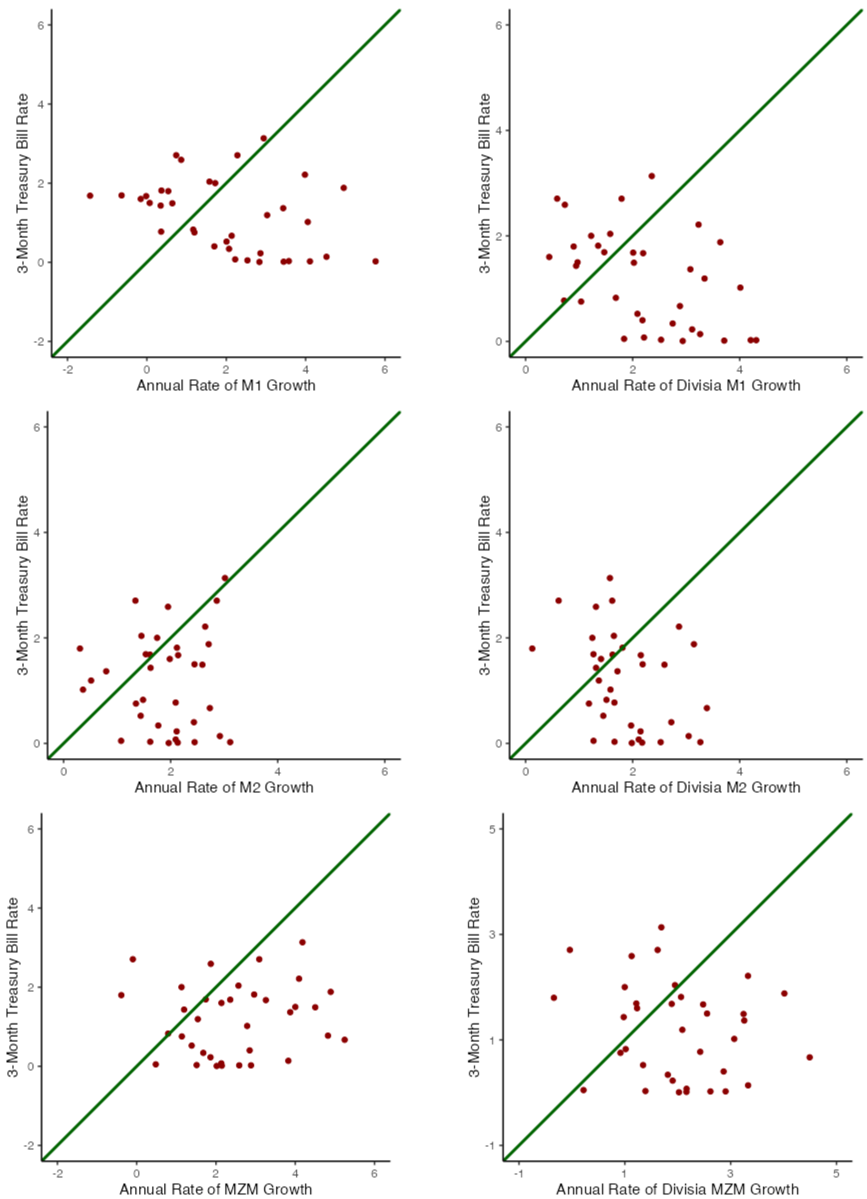
Figure A.15: Scatter Plots of Modified Hamilton Filtered Short-term Interest Rate and the Money Growth; 1968–83



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

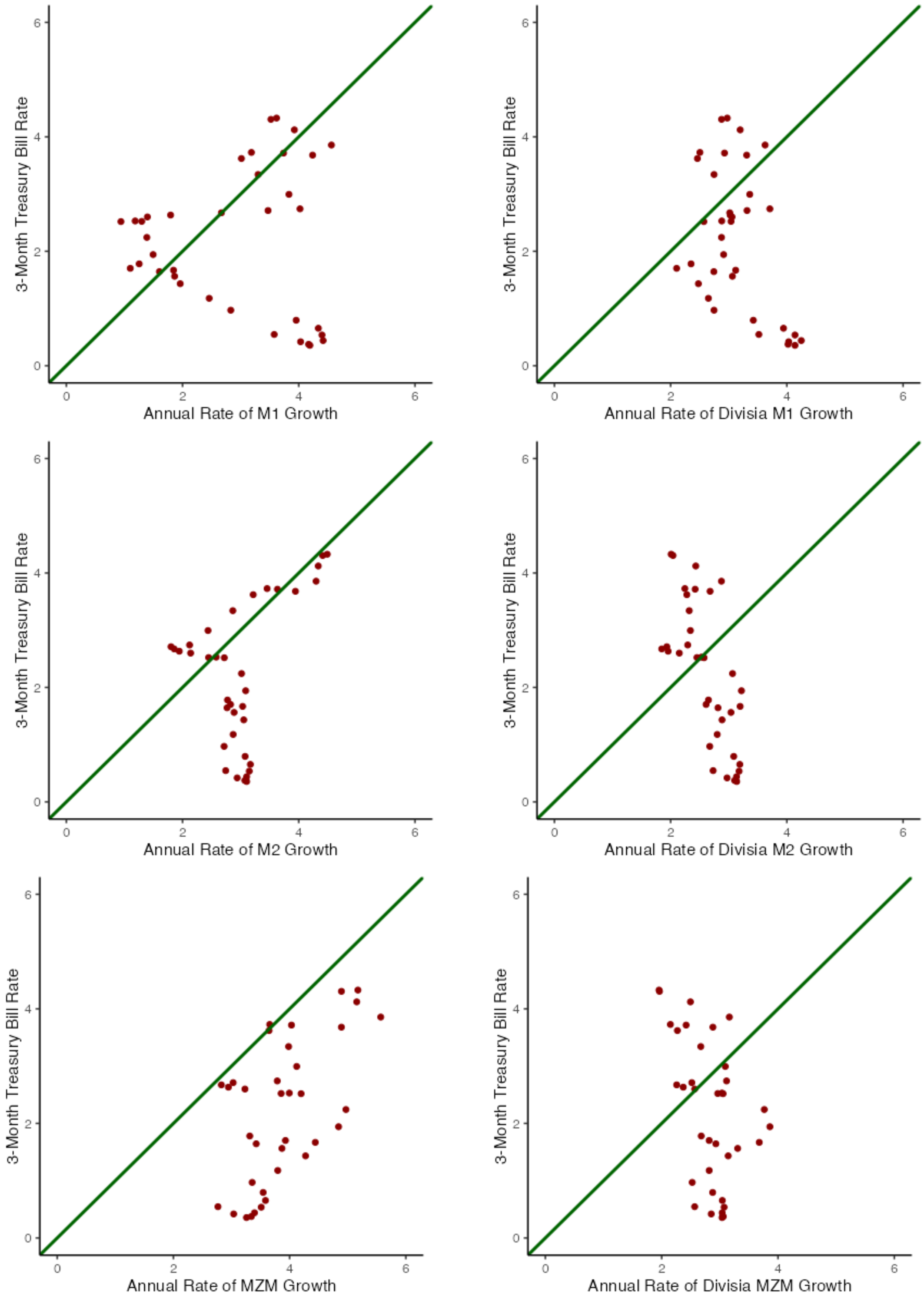
A.4 The Recent Subsample: Smoothed Data for 1984–2019

Figure A.16: Scatter Plots of EWMA Filtered Short-term Interest Rate and the Money Growth ($\beta = 0.5$); 1984–2019



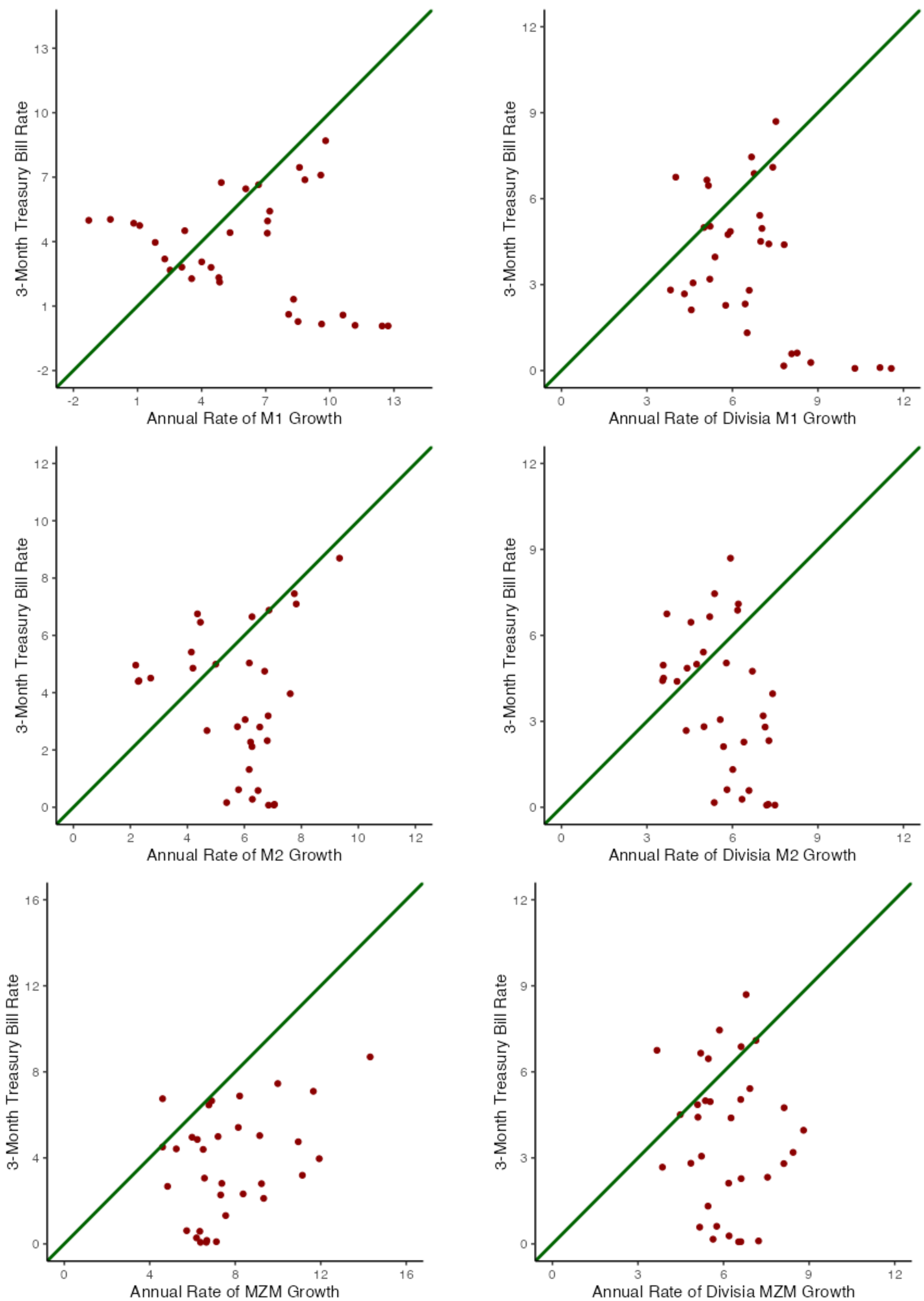
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.17: Scatter Plots of EWMA Filtered Short-term Interest Rate and the Money Growth ($\beta = 0.95$); 1984–2019



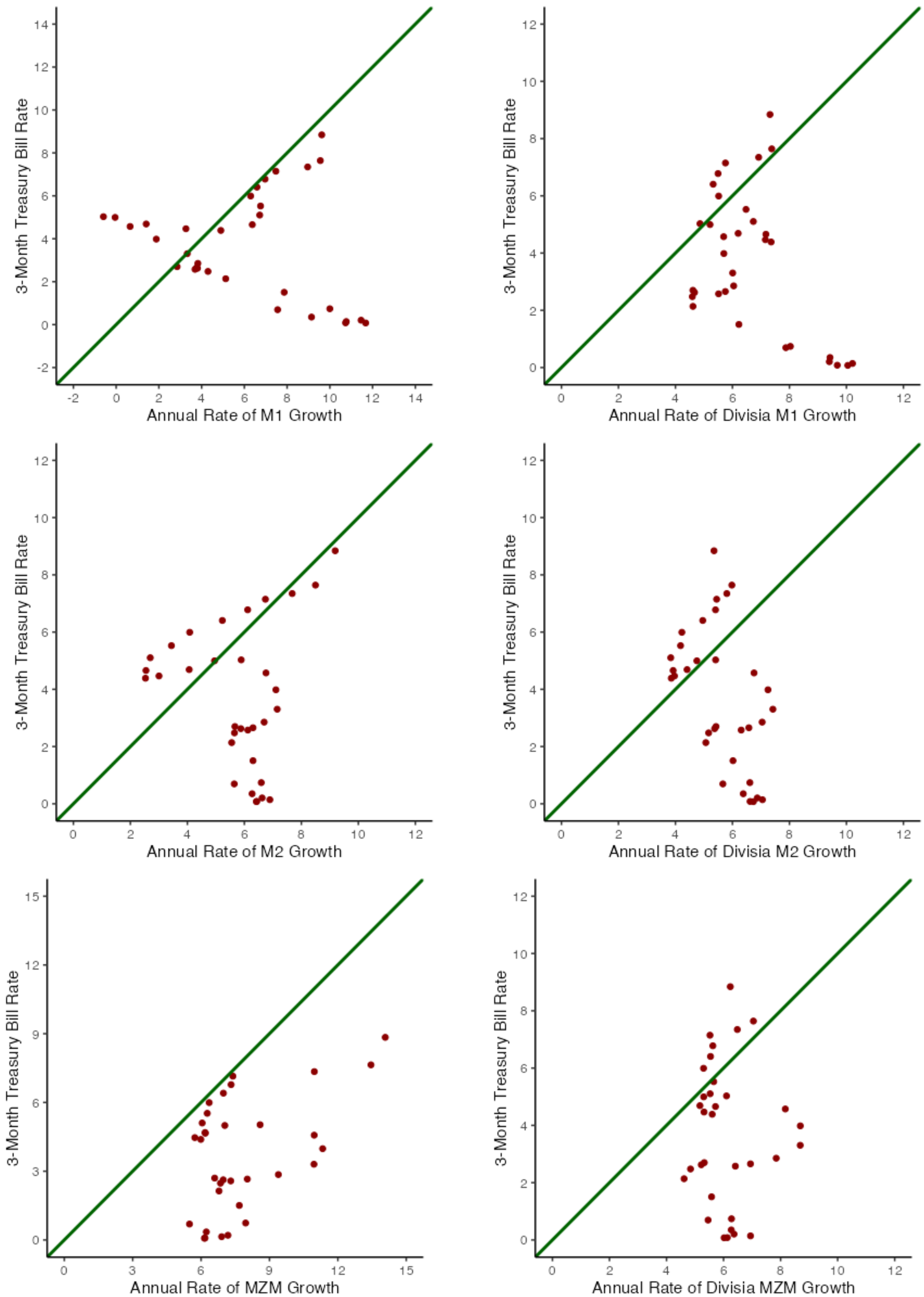
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.18: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(6) & max(32)); 1984–2019



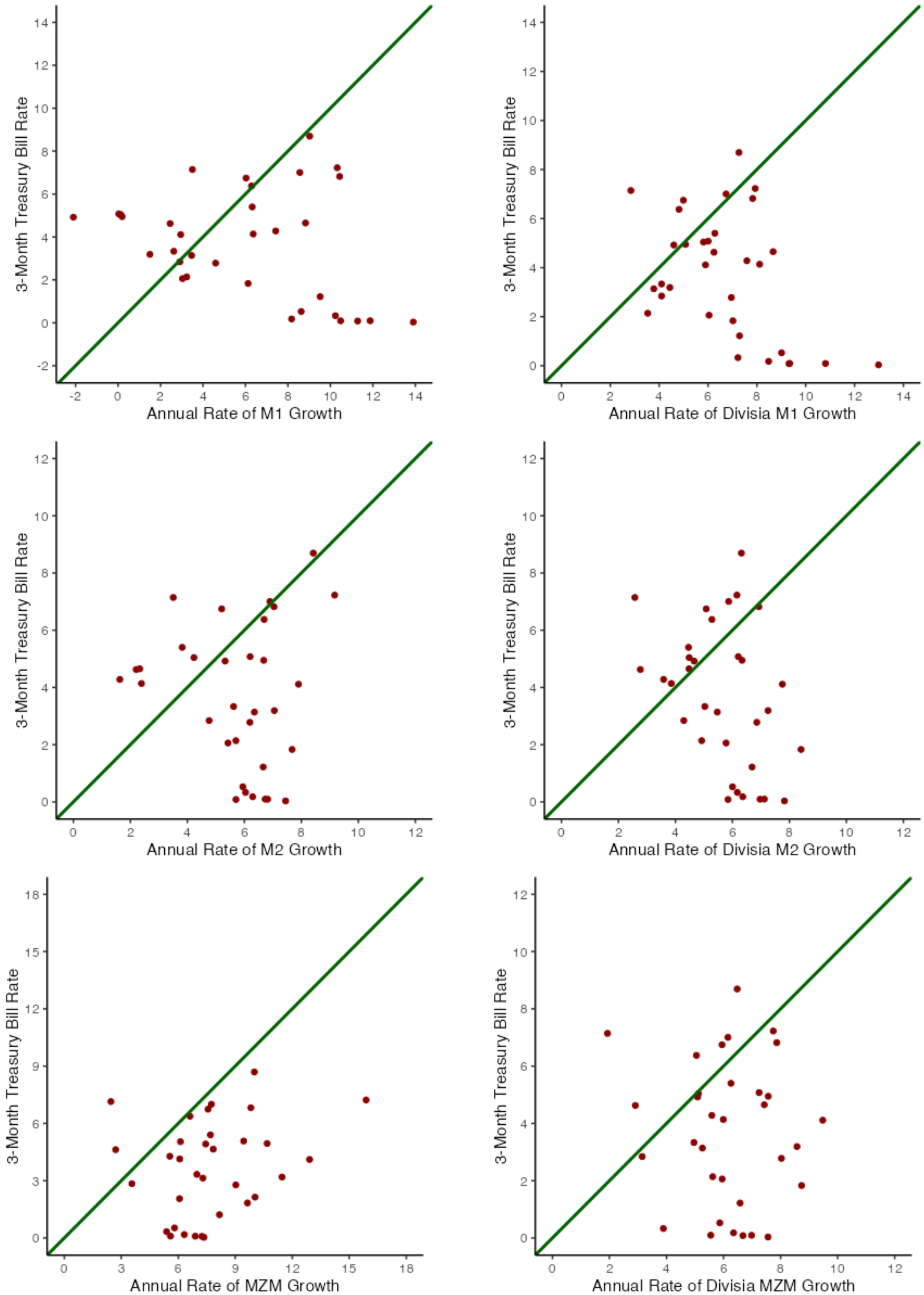
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.19: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(2) & max(60)); 1984–2019



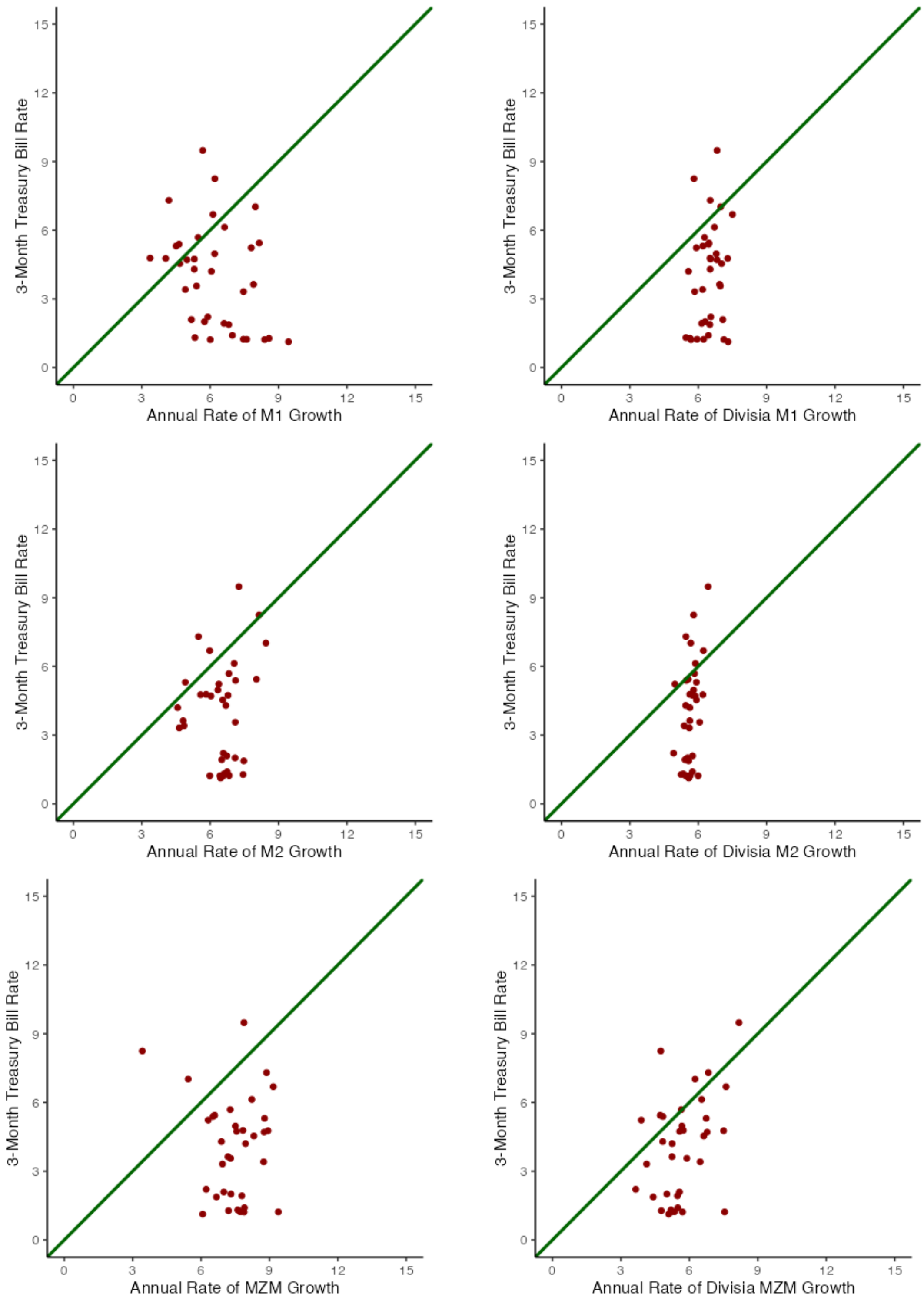
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.20: Scatter Plots of BK Filtered Short-term Interest Rate and the Money Growth (min(8) & max(30)); 1984–2019



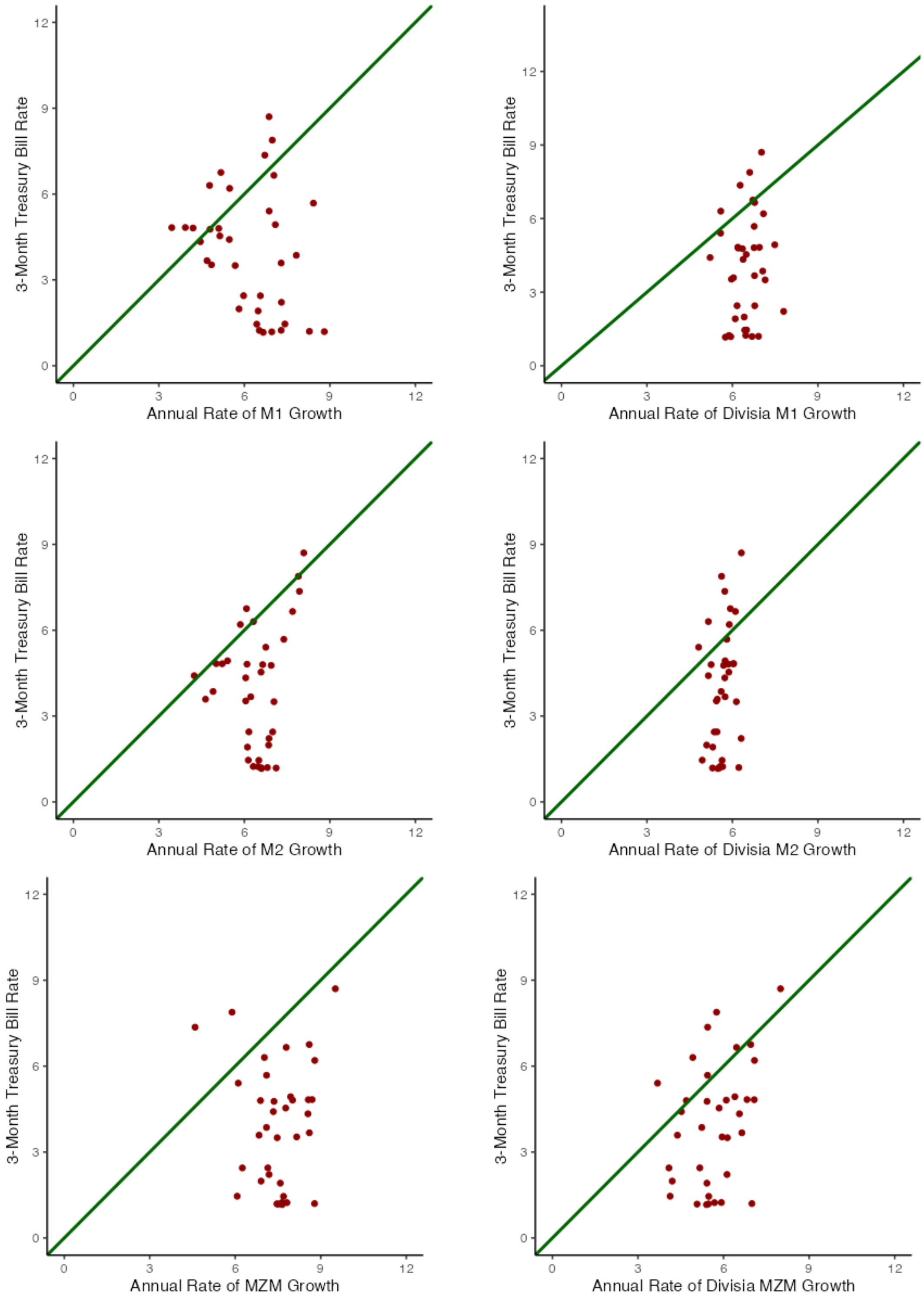
Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.21: Scatter Plots of Hamilton Filtered Short-term Interest Rate and the Money Growth; 1984–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line

Figure A.22: Scatter Plots of Modified Hamilton Filtered Short-term Interest Rate and the Money Growth; 1984–2019



Notes: Results are reported for the second quater of each year and the *green-solid line* is the 45 degree line